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Issues paper for Pre-consultation

Determination of the risk free interest rate term structure for Solvency II

1 Introduction and Scope

The Solvency II regulation provides that EIOPA may develop draft technical standards in the areas specifically set out in the relevant legislation to be submitted to the Commission for adoption. Areas that are selected are those in which consistent technical rules will contribute significantly and effectively to the achievement of the objectives of the relevant legislation, building a single set of harmonised rules and avoiding unduly complicated regulation and enforcement.

One of the areas where EIOPA is developing a technical standard is the determination of the risk-free interest rate term structure to be used by all (re-) insurance undertakings for the purpose of calculating technical provisions as required by article 77 of the Directive. The technical standard is intended to further develop, specify and determine the conditions of application of the level 2 measures around the determination of the risk free interest rate term structure.

EIOPA is currently in the process of developing the implementing technical standards (ITS). This document is intended to both give stakeholders an early opportunity to provide comments on EIOPA's development of the risk free term structure and EIOPA to use the stakeholders' input to develop the ITS for consultation in Q3 2012.

Please note that the text in this document goes beyond what will be covered in an ITS and hence is presented differently than in the forthcoming consultation on the implementing standards. In addition, EIOPA also asks the industry on their views on some procedural aspects.

It has to be emphasised that although agreement has been reached among member states on many aspects of the risk free term structure, a number of issues still need further thorough considerations before they can be incorporated into implementing technical standards. Where further decisions still have to be made, this will be highlighted in the text.

The document is organised into sections that cover the main cornerstones of the basic risk free rate term structure. Following this brief introduction, some information on the legal background is provided. The section three covers the selection and treatment of data that will form input factors into the eventual risk free term structure. In section four it is explained how the curve is fitted and how extrapolation beyond accessible and relevant data is done. The details in these sections should provide sufficient information to derive the basic risk free term structure. Due to significant on-going developments, section five and six on the countercyclical premium and the matching premium respectively only function as a placeholder. Finally, process related aspects, which will not be covered in the ITS, are discussed and the main input factors for the exemplary risk free term structures are provided. The annex provides additional background information that is intended to bring the information in the main body of the document into perspective. It also highlights areas where alternative views exist and where EIOPA invites stakeholders' views. Where this is the case a reference is made in the core text. The zero coupon curves for Euro, GBP and USD are provided as a proof of concepts settled in the current issues paper.

The subsequent table summarises the respective modules of the risk free term structure derivation, where they are discussed and where EIOPA is specifically but not exclusively seeking comments from stakeholders.

Table: specific aspects for the determination of the basic risk free interest rate

Relevant aspect	Ref.	Specific questions raised with stakeholders
Publication of curves	7 7 7	• Do you agree with the approach taken to group currencies into different buckets of publication (i.e. more and less frequent publication, no publication)? • Do you agree with the selection of currencies? Please indicate which currencies you think need to be in the "high frequency bucket" • Please state and explain your preference in terms of frequency of publication, e.g. daily, monthly, etc.
Underlying data - Reference instruments - bid/mid/ask price - credit and basis risk adjustment	3.1.1 3.2.1 3.2.2	• EIOPA invites comments on how to deal with switches of reference instruments for only few data points • Do you agree with the use of swap bid rates? • EIOPA welcomes views on how to measure the basis risk adjustment. • EIOPA would like to hear stakeholders' views on the approaches taken and assumptions made on credit risk adjustment.
Treatment of pegged currencies	3.2.3	• EIOPA welcomes views on the adjustment for pegged currencies.
ADLT assessment, Last liquid point and entry point to extrapolation	3.1	• Do you agree with the general approach taken? • EIOPA invites comments on the two suggested alternatives to find the entry point to extrapolation • EIOPA also invites comments on outlined measures to assess the ADLT requirements and further possible approaches • Do you agree with the approach to assess the eventual entry point to extrapolation?
Extrapolation and interpolation methodology - speed of convergence	4	• Do you agree with the methodology chosen for extra- and interpolation? • Do stakeholders think that artificial volatility in the extrapolation is a concern and if so how should it be addressed? • For reasons of pragmatism, EIOPA suggests ap-

- Ultimate forward rate	4.2.1	plying one single rate as UFR for all currencies instead to for instance three different buckets as applied in QIS5. Do you agree with this simplifying approach? If you consider that there is a need for additional buckets, please also state why by indicating the implications of a single UFR.
Countercyclical premium	5	Please note that this subject is still subject to ongoing discussions and is therefore only included as a placeholder
Matching premium	6	Please note that this subject is still subject to ongoing discussions and is therefore only included as a placeholder

2 Legal background and objectives

According to article 77(2) of the Directive, *"the best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure."*

Following this, Article 86 of the Directive states that *"the Commission shall adopt implementing measures laying down … the relevant risk-free interest rate term structure to be used to calculate the best estimate referred to in Article 77(2)*.

Level 2 will set out the basic framework for the derivation of the relevant risk-free term structure including requirements on the basic risk free interest rate term structure (non-extrapolated part of the curve), the extrapolation approach to be applied as well as specific requirements on the determination and application of a counter-cyclical premium. An overall consistent approach to the different components of the risk free term structure should be aimed at avoiding that assumptions on which the curve is based differ between e.g. the non-extrapolated and the extrapolated part of the curve.

The risk-free interest rate term structure should be determined on the basis of a holistic and consistent approach to the setting of all assumptions and parameters on which the curve is based ensuring consistency over time and avoiding artificial volatility of technical provisions and eligible own funds in excess of the capital requirements.

To ensure a holistic and consistent approach in the determination of the risk free term structure, it is considered important to set out the main objectives that the curve, and thus the methods set out in this paper for the determination of the curve, should be geared to.

The following are considered to be important objectives that should be kept in mind when developing a holistic approach to the determination of the interest rate term structure:

1. <u>Continuity:</u> The interest rate term structure should be theoretically and economically sound, ensuring an arbitrage-free interest rate curve which is continuous.
2. <u>Smoothness:</u> The methodology to determine the interest rate term structure should provide for smooth interest rate curves.
3. <u>Robustness and stability:</u> The model should produce curves that reflect only the volatility in the market and provide for a reasonable variation over time.
4. <u>Accuracy:</u> The methodology should provide a good fit to the liquid market data (accuracy).
5. <u>Transparency and practicability:</u> The methodology should be fully transparent, easy to apply and accessible to undertakings to ensure applicability in times where EIOPA does not publish the interest rate term structure.
6. <u>Incentives:</u> The methodology should not give inappropriate risk management incentives.

Notwithstanding the detailed objectives of determining the risk free term structure, the calculation of technical provisions should be market consistent (recital 54, Directive 2009/138/EC).

3 Determination of the basic risk-free term structure

The relevant risk free interest rate term-structure is composed of the rates of a basic risk-free interest rate term structure plus, where applicable, a counter-cyclical premium or matching premium for each currency and maturity. To determine the basic risk-free term structure, considerations have to be made in respect of the availability and the relevance of the data. In addition, those data have to be adjusted for the inherent credit risk or take into account the peg to another lead currency, if applicable.

3.1 Selection criteria for data and determination of entry point to extrapolation

The assessment and derivation of the liquid part of the risk free term structure can be broken down into three (or alternatively four) integral steps.

The starting point of the assessment is to check in a first step the availability of swaps and government bonds for each currency and the transparency of the relevant market in general[1]. For each currency, this will identify the longest maturity for which swap and/or government bond rates are potentially available, and will thus set an absolute upper bound on the entry point into extrapolation.

In a second step, the available market data for the reference instruments will be examined on whether and up to which maturity they fulfil the criteria of having an active, deep, liquid and transparent (ADLT) market. This assessment is both a refinement of the availability assessment and a method to derive the last liquid point, i.e. the longest maturity at which ADLT requirements are still fulfilled, with respect to the market for reference instruments. This may be denoted by $LLP^{reference}$.

In the next step(s), the ADLT assessment has to be carried out for the overall bond markets (i.e. sovereign and corporate bonds), supported by an assessment of the ability of insurers to match their insurance liabilities with bonds. Where it would no longer be possible for insurers to match insurance liabilities with bonds of the same currency, this should be reflected in the determination of LLP^{bonds}

Combining all the steps of the assessment, the overall last liquid point or entry point to extrapolation, is determined as:

$$\mathbf{LLP = min(LLP^{reference}, LLP^{bonds})}$$

EIOPA has not reached an agreement on whether the LLP^{bond} assessment should be undertaken as one or two separate steps. In order not to favour one over the other, both approaches are described in annex 9.2. EIOPA strongly welcomes the

[1] It is not expected that this availability assessment has to be repeated every time a risk free term structure is derived. However, it has to be taken into account that financial markets develop over time and so does the availability of financial instruments.

views of stakeholders. EIOPA also invites comments on the methods for this assessment (cf. 9.3)

In the following sub-sections, these steps are supported by further considerations on:

- General methodology applied to carry out the ADLT assessment (cf. section 3.1.1)
- The choice between swaps and government bonds, as relevant for steps 1 and 2 (cf. section 3.1.2)
- The concretion of the general criteria of an active, deep, liquid and transparent market (cf. section 3.1.3)

For the aforementioned reasons, the assessment of the degree to which insurers may match their liabilities with bonds and the eventual determination of the entry point to extrapolation are covered in the annex.

3.1.1 General considerations on assessment methodology for ADLT criteria

The risk free interest rate term structure should be:

- Calculated *separately for each currency and maturity*, based on information and data *relevant for that currency and maturity*
- Determined in a *transparent, prudent, reliable* and *objective* manner

Logically, this suggests that each constituent process in the construction of the risk free term structure must therefore adhere to these principles.

The requirement to determine the structure in an *objective* manner suggests that for each currency, the assessment of the maturities for which the swap and/or government bond market or the market for bonds as a whole can be considered ADLT should be performed in as objective a manner as possible. This suggests that to the extent possible the methodology should be based on quantitative factors. Expert judgement and qualitative factors should also be part of the process but this should be based on objective and transparent criteria.

EIOPA must publish, for each currency, the methods used to determine the longest maturities for which interest rates can be observed in liquid markets. Combined with the requirement that the term structure be determined in a *transparent* manner, this means that the criteria for determining the ADLT condition, both qualitative and quantitative, should be published regularly along with any relative weighting assigned to each criterion. The need for transparency also extends to the inputs to the methodology.

The requirement that the term structure be determined in a *reliable* manner suggests that the methodology for identifying ADLT markets for a given maturity should not be unduly sensitive to its inputs (since this would introduce additional volatility and unpredictability). In particular, given its influence on the overall volatility of the risk free curve (and therefore on Technical Provisions), the methodology should be sufficiently robust that the assessment of the Last Liquid Point (LLP) returns consistent answers in normal market conditions. This robustness seems more likely to exist where the methodology takes into account all relevant sources of reliable information and where more than one measure is used to arrive at the overall determination (cf. annex for an assessment of ADLT measures).

The requirement for *prudence* suggests that where market data is used to set the risk free rate, there should be a high degree of confidence that this market data is reliable in that it has come from a market that is active, deep, liquid and transparent. To achieve a high degree of confidence that the conditions are met, the assessment criteria for each condition (active, deep, liquid, transparent) should be calibrated to strike an appropriate balance between strictness and pragmatism[2].

In summary to meet the requirement that the term structure is determined in an objective, reliable, prudent and transparent manner, the method for assessing ADLT market conditions should be:

- Based – wherever reasonable - on quantitative criteria;
- Automated as far as possible and only allow expert judgement where this is based on transparent and objective criteria;
- Published by EIOPA, along with all assessment criteria and relevant weightings;
- Robust, in the sense that it delivers relatively stable outcomes in normal market conditions;
- Based on a range of the main publicly available and relevant sources of information; rather than on any one single measure;
- Calibrated to give a high degree of confidence that all market data included in the construction of the risk free term structure has indeed come from ADLT markets;
- Revised periodically if appropriate.

It has to be emphasised that quantitative criteria may vary considerably in absolute terms. This is largely a result of the size of the markets and hence it would for instance, not make sense to use the same volume thresholds to assess the depth of different financial markets. Furthermore, different indicators have to be assessed in conjunction as a measure looked at in isolation may not always provide sufficient information of whether a financial instrument is liquid or not.

At a future point in time, developments such as the introduction of EMIR[3], or innovations by market data providers, may mean that a reliable centralised source of trade data for OTC instruments becomes available. This will allow a more thorough assessment of whether and up to what point financial instruments fulfil the

[2] For instance, it is undesirable that the criteria be so inflexible that they can result in the rejection of data from markets that are generally accepted as liquid.

[3] European Market Infrastructure Regulation, which will mandate the central clearing of all standardised OTC derivative contracts as well as introducing new collateral requirements for non-standard contracts.

ADLT requirements. EIOPA will continue to monitor developments in this area and seek ways to supplement the ADLT assessment as appropriate.

For the purpose of the pre-consultation and the construction of the curves (EUR, GBP, USD) the QIS5 methodology will be used as the basis, with the elements of the analysis (e.g. contributor counts) being updated to reflect prevailing market conditions.

Do you agree with the general approach? EIOPA also welcomes views on what additional data may be considered for an ADLT assessment and in particular how/where to set the thresholds. In the Annex, a number of measures are provided that are considered for an ADLT assessment (and the potential development of a scorecard).

3.1.2 Choice between swap rates and government bond rates as reference instruments

The basic risk-free interest rate term structure should be derived, where appropriate, on the basis of interest rate swap rates adjusted to take account of the credit risk and the basis risk of the swaps. However, where swap rates are unavailable or the rates are not available from active, deep, liquid and transparent markets, government bond rates shall be used instead where these can be derived from active ADLT markets. Both, swaps and government bonds have to be adjusted for credit risk[4]

The following decision tree can be used for Solvency II purposes:

1. *For each currency and maturity where swaps exist from markets which satisfy the ADLT criteria, the basic risk-free interest rate applicable to the valuation of a liability should be based on the swap curve appropriately adjusted to remove credit risk and the basis risk of the corresponding interest rate swap.*
2. *Where swaps do not exist from markets which are active, deep, liquid and transparent, the basic risk-free interest rate applicable to the valuation of a liability should have reference to the government curve in that currency adjusted for the credit risk of the corresponding government bond, provided that the market for government bonds is also, active, deep, liquid and transparent.*
3. *If for some maturities there is no active deep, liquid and transparent swap market but the government bond market fulfils the criteria, then the government bond will be used as the reference instrument for those maturities*[5]*.*

[4] Please refer to detailed description of credit risk adjustment in respective sub-section.

[5] This point follows from the previous two bullet points. However, for transparency purposes it was stated explicitly

A combination of both swaps and government bonds can be used to construct the curve for a single currency[6]. This means that for currencies where government bonds are available at longer maturities than are swaps, an ADLT assessment for both types of instruments is required because this will determine the points at which the curve switches from being based on one type of instrument to the other.

There are obvious practical difficulties with implementing this approach (e.g. the difficulty of ensuring a smooth curve when combining data from two distinct markets). This may be of particular relevance when government bond maturities (that fulfil the requirements) exceed those of swaps by only a short period, one year say. This will potentially create a hump shaped curve. Introducing a combination of swaps and government bonds may have implications on the smoothness and robustness of the determination of the curve. Hence, EIOPA will very carefully look into this issue over the coming months.

In addition, to the aforementioned issue, one may also consider the possibility that there may be gaps in the liquidity of the swap curve, thus theoretically justifying multiple switches between swaps and government bonds. However, apart from not being aware of any example in practice, EIOPA considers that such a multiple switch of instruments can lead to considerable technical difficulties.

EIOPA invites comments on how to deal with the aforementioned issues.

EIOPA undertook a high level stock taking exercise (cf. also table in Annex) to understand the relevance of this challenge for the 35 chosen currencies. The exercise showed that for a number of currencies the maximum swap term available (ADLT was not assessed at this stage) is ten years or less. The Brazilian Real (BRL) has, for instance, a maximum available swap term of 5 years but the maximum term for generic government bonds is 30 years. The situation is similar for Chinese Yuan with ten years versus 49.5 years. One may assume, though that in particular for those two currencies the situation will develop significantly over the near future.

For some currencies no swap data could be sourced at all, i.e. Algerian Dinar, Argentine Peso, Croatian Kuna, Latvia Lats, Lithuanian Litas[7], Tunisian Dinar and Venezuelan Bolivar.

EIOPA will further look into this issue both in order to understand the practical relevance and to understand better the implications. In practice, though, most relevant currencies will not be affected by this particularity.

3.1.3 Concretion of the criteria of an active, deep, liquid and transparent market

[6] By way of example we can consider a notional currency for which the swap market is ADLT up to 5 years' duration but the government bond market is ADLT out to 30 years' duration.

[7] LVL and LTS are both pegged to the Euro and are thus treated as pegged currencies.

The rates of the basic risk-free interest rate term structure shall be reliably determined based on financial instruments traded in markets that are active, deep, liquid and transparent. The financial instruments must be traded in active markets as defined in international accounting standards[8] and, in addition, meet all of the following general criteria of depth, liquidity and transparency:

(a) *Transactions involving a large quantity of financial instruments can take place without significantly affecting the price of the instruments (deep),*

(b) *Financial instruments can readily be converted through an act of buying or selling without causing a significant movement in the price (liquid),*

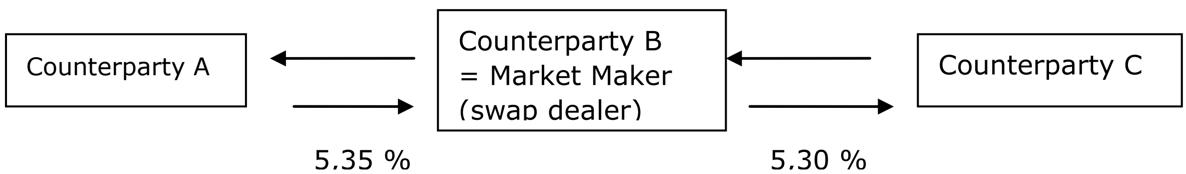
(c) *Current trade and price information is readily available to the public, in particular to the undertakings (transparent).*

The criteria for an active market, as defined in international accounting standards, is also used in the context of the valuation of assets and liabilities other than technical provisions.

3.2 Treatment of data used for derivation of risk free term structure

3.2.1 Bid, mid and ask rates of swaps

Swap data are quoted as bid, mid and ask rates by data providers and are assessed from swap rates or prices delivered by market makers. Market makers usually quote two swap prices – a lower and a higher. The lower rate is the rate they are willing to pay as fixed rate (quoted as bid rate) and the higher rate is the rate they ask when they receive the fixed rate (quoted as ask rate).



A matched pair of swap transactions (same maturity)

[8] In IFRS the term "active market" is used. The following definition of an active market can be found in IAS.36.6, IAS.38.8 or IAS.41.8:
An active market is a market where all the following conditions exist:
- a. *The items traded within the market are homogeneous;*
- b. *Willing buyers and sellers can normally be found at any time; and*
- c. *Prices are available to the public.*

According to IFRS 13 (May 2011) an active market is a market in which transactions for the asset or liability take place with sufficient frequency and volume to provide pricing information on an ongoing basis. The IASB treats this definition as consistent with the above.

Insurance companies will typically be in the position of Counterparty C, i.e. fixed-rate receivers. The bid rate is the highest rate they can earn when swapping the floating return on a notional. By taking the mid-market prices as basis for the risk-free discounting, the liabilities would be understated.

For that reason, EIOPA suggests to use the swap bid rate as basis for the valuation of insurance liabilities.

Such an approach would also be in line with current accounting standards as prescribed in IAS 39. However, we note, that IFRS 13 will supersede IAS 39, once it is endorsed by the EU. IFRS 13 allows using mid-price valuation, which would then carry across from the accounting onto the Solvency II balance sheet.

The change in accounting should not alter the approach taken to the determination of risk free rates. However, it should be emphasised that the sole focus on liquid instruments implies, in general, small bid-ask spreads and hence only little difference between bid and mid rates.

Do you agree with the use of swap bid rates instead of mid rates[9] *(as used under QIS5), taking account of the arguments made?*

3.2.2 Adjustment for credit risk

The financial instruments used to derive the basic risk free interest rate term structure, i.e. the reference instruments need to be adjusted for credit risk, and if the reference instrument is based on interest rate swaps it needs also to be adjusted for basis risk. The credit risk adjustment should be performed both when the basic risk free interest rate term structure is based on interest rate swaps and when it is based on government bonds. The credit risk adjustment shall reflect current market conditions.

Credit and basis risk adjustment of interest rate swaps

Interest rate swaps are potentially exposed to two different kinds of credit risk:

- A counterparty credit risk due to the risk of default of the counterparty where this would lead to a loss because of a positive market value.
- The credit risk associated with earning the floating rate used in the interest rate swap. Typically the floating rate is based on 3 or 6 months floating rates. This risk has historical been negligible, however, since 2008 this is no longer the case.

[9] Please note that the difference between bid and mid-market rate for the major currencies tends to be low but is inversely related to the liquidity of the instrument. Thus, the spread can be more material for less developed countries.

It is EIOPA's position that counterparty credit risk is neglected on the basis that ISDA/CSA arrangements reduce this risk to a minimum by requiring collateral. This is similar to the approach taken under QIS5.

The adjustment should therefore only take into account the credit risk associated with the floating leg. The spread resulting from credit risk can be approximated by e.g. the spread between the reference floating leg of the swap and an overnight rate (Eonia for EUR) or the spread between the reference floating leg and a secured lending rate with same maturity (Eurepo for EUR).

Such an approximation will imply that the same absolute adjustment will be used on all maturities (on the non-extrapolated part of the interest rate term structure), implicitly assuming that credit risk is unaffected by the tenure of the instrument. EIOPA acknowledges that this is a simplification since the shape of interest rate curves can affect the size of the spreads. Nevertheless this simplification is necessary primarily due to lack of data to estimate a more accurate maturity-dependent adjustment.

The approximated spread is widely known to include credit risk as a major component, however, in times of liquidity crises in interbank markets the liquidity risk component can become the more important component.

In relation to the credit risk adjustment neglecting the liquidity risk component could have influence on the interest rate term structure in crisis situations. The risk is that in case of an active countercyclical premium (CCP) because of illiquidity the adjustment for credit risk would to some extent counterbalance the CCP. As a result the CCP would have limited effect.

EIOPA's suggestion is to use a simplistic approach that assumes the liquidity component is negligible in normal situations where the CCP is not activated. When the CCP is activated a separate calculation of the liquidity component will be done[10].

It is a requirement that the proposed method for adjustment must be applicable for all relevant currencies. The proposal for QIS5 was based on an estimate from four main currencies (EUR, GBP, USD and JPY) which then lead to a proxy applied for all currencies.

EIOPA is proposing to use a similar method for risk-free interest rate term structure including the use of same data sources; however, a final decision has not been made. The adjustment for credit risk will hence be the same for all currencies where swaps are used to derive the curve. Interest rate swaps are not by definition currency specific in the sense that a EUR swap can be based on e.g. LIBOR or EURIBOR as the reference floating rate. To achieve consistency across markets it is found sensible to use the same adjustment for all currencies.

EIOPA is interested to hear stakeholders view on the choice of data to calculate the adjustment for credit risk. Do stakeholders see better data than what was used in QIS5?

[10] The CCP is still under development. So, no details can be provided at this stage.

Another condition in the draft implementing measures is that the adjustment shall reflect current market conditions[11]. EIOPA considers at least two relevant methods in order to fulfil the requirement that the adjustment shall reflect current market conditions:

- Use the actual observed spread as the adjustment
- Use a moving average of observed spreads over a given time period as the adjustment

Choosing the first option will introduce more volatility in the interest rate curve compared to the smoothing mechanism inherent in the second option. EIOPA has no strong position on this, but tends to prefer a moving average based on the assumption that undertakings are not able to hedge the volatility introduced with the use of actual observed spreads.

EIOPA is interested to hear stakeholders view on whether the volatility introduced by using the actual observed spreads is hedgeable for undertakings as well as stakeholders preferred method.

To conclude EIOPA proposes a credit risk adjustment based on the following key points:

- It reflects the credit risk related to the interbank market (earning the floating rate).
- It is based on the spread between short term unsecured and secured lending.
- It will be the same for all maturities of the non-extrapolated part of the curve as well as for all currencies.
- It is not corrected for liquidity risk as long as the market situation is normal i.e. no CCP. The liquidity risk component will be explicitly calculated once the CCP is activated.
- It is based on a moving average of actual observed spreads

EIOPA welcomes stakeholders' views of how to measure and interpret the basis risk, in this context defined as the risk of loss or of adverse change in the financial situation of the holder of the interest rate swaps, resulting from the mismatch between the cash in-flows and the cash out-flows of the interest rate swap.

Credit risk adjustment for government bonds

EIOPA is currently still considering how to adjust for credit and basis risk when the interest rate curve is derived from government bonds. This will only affect those currencies which only have a sufficiently mature government bond market;

[11] Under QIS5 this was not a requirement.

such that the risk free term structure will have to be built from government bonds rather than swaps.

The technical approach to measuring credit risk on sovereigns is to look at CDS premiums on government bonds. This is based on the no arbitrage relation that the CDS premium expresses the difference between the yield of a risk free bond and a risky bond. The relation has been tested in several studies and has proven to be rather strong. Some studies show, however, that recently the CDS premium has included a small basis which is not related to credit risk of the sovereign.

Several more simplistic approaches are considered as alternatives. As an example one could base the adjustment on the rating of the sovereign. The starting point for such a calibration could be the CDS approach but would of course be partly subjective. One may also question whether this approach reflects current market conditions.

For currencies where swaps exist for shorter maturities than government bonds an alternative is to use the spread between government bonds and swaps corrected for credit and basis risk as an adjustment. As an example for a given currency swaps are used to derive the interest rate curve until 10 years after which government bonds are used until 15 years. In this case the credit risk adjustment would be based on the observed spread between 10Y government bonds and 10Y swaps (taking account of the credit risk and basis risk adjustment for swaps, 10bp say. In this case the adjustment would be swaps minus government bonds – 10bp). This spread would then be used to adjust maturities from 10 to 15 years.

The advantage of this approach is that the interest rate term structure would not be discontinuous at the maturity where government bonds are used instead of swaps.

In all cases the adjustment is proposed to be the same for all maturities.

EIOPA is interested in stakeholders view on the adjustment for credit risk of government bonds in particular whether the simplistic alternatives are reasonable.

3.2.3 Treatment of currencies pegged to the euro

Relevant currencies

It was decided that currencies pegged to the Euro shall be limited to the following:

Eligible currencies participating in the European Exchange Rate Mechanism (ERM II):

- Danish Krone (DKK)
- Lithuanian Litas (LTL)
- Latvian Lats (LVL)
- Bulgarian Lev (BGN)

Eligible currencies where a decision from the Council recognises pegging arrangements between this currency and the euro

- Comorian Franc (KMF)
- Central African CFA Franc (XAF)
- West African CFA Franc (XOF)

This is explained by the following arguments:

The Bank of England is party to the Agreement of 16 March 2006 (2006/C 73/08[12]) between the European Central Bank and the national central banks of the Member States outside the euro area laying down the operating procedures for an exchange rate mechanism in stage three of Economic and Monetary Union ("ERM II"), but has not fixed a central rate nor a fluctuation corridor with the Euro, and therefore the British Pound cannot be considered as pegged to the Euro. The same situation applies to the Central Banks of Poland, Sweden, Hungary and the Czech Republic.

The Bulgarian National Bank is however party to the Agreement of 14 December 2007 (2007/C 319/04)[13], which updates the Agreement of 16 March 2006, and can therefore now be considered party to ERMII, and has fixed its currency in a pegging arrangement by the 1997 Law on National Bulgarian Bank. The Romanian Central Bank is also a new party to this Agreement, but has not fixed its currency in a pegging arrangement.

Therefore it can be said that only Denmark, Lithuania, Latvia and Bulgaria fully participate in ERM II.

Pegging of the CFA Franc and the Comorian Franc is recognised by the Council Decision (98/683/EC)[14] of 23 November 1998 concerning exchange rate matters relating to the CFA Franc and the Comorian Franc.

Pegging of the Cape Verde Escudo to the Euro is recognized by the Council Decision (98/744/EC)15 of 21 December 1998 concerning exchange-rate matters relating to the Cape Verde escudo [Official Journal L 358 of 31.12.1998]. This Council Decision indicates that the convertibility of the Cape Verde escudo is ensured by a limited credit facility provided by the Portuguese government and that the Portuguese government has given the assurance that the agreement with Cape Verde has no substantial financial implications for Portugal.

The credit line open for strengthening of exchange reserves is however limited to 5,500 million or 9,000 millions of Portuguese Escudos as per the referenced article in the Council Decision (Acordo de cooperação cambial entre a República Portuguesa e a República de Cabo Verde. (Decreto nº 24/98 de 15 de Julho 1998))16.

[12] http://www.ecb.int/ecb/legal/pdf/c_07320060325en00210027.pdf
[13] http://www.ecb.int/ecb/legal/pdf/c_31920071229en00070009.pdf
[14] http://eur-lex.europa.eu/smartapi/cgi/sga_doc?smartapi!celexplus!prod!DocNumber&lg=en&type_doc=Decision&an_doc=1998&nu_doc=683
[15] http://eur-lex.europa.eu/smartapi/cgi/sga_doc?smartapi!celexplus!prod!DocNumber&lg=en&type_doc=Decision&an_doc=1998&nu_doc=744
[16] http://www.gddc.pt/siii/docs/dec24-1998.pdf

Therefore the financial resources of the parties that guarantee the pegging might jeopardize the robustness of the pegging arrangement, which is a risk identified in Article 172.5 CR1.

Approach to calculation

The adjustment to be performed when Euro denominated financial instruments are used to derive the risk-free interest rate term structure for currencies pegged to the Euro should in principle approximate the premium that investors require to hold financial instruments in the local currency. This premium would include payment for changes in the exchange rate which are within the bands of the pegging arrangement.

There are at least three possible solutions to approximate the premium by looking at financial instruments. These include:

- Spreads between local government bonds and AAA rated government bonds from the Euro-area e.g. German government bonds
- FX forward contracts used to hedge the relevant FX risk
- Cross currency swaps

The choice of adjustment should take into account two criteria, namely

- undertakings possibility to hedge the adjustment and
- liquidity in the relevant financial instruments used for the adjustment.

The government bond spread method will probably have sufficient liquidity for certain maturities only. The lack of liquidity would almost always be the case for the local government bonds. This will imply that undertakings are not able to hedge changes in technical provisions due to changes in the spread since insufficient volumes of local government bonds exist. Such a situation has been experienced for at least one currency pegged to the Euro.

FX forwards are usually very liquid, however, the contracts are typically short term and the adjustment to all maturities would therefore be based on contracts with maturities less than 1 year.

Cross currency swaps are in general not sufficiently liquid to derive such an adjustment without running the risk of impact on pricing since undertakings will try to hedge the adjustment in this market.

In conclusion it is arguable whether any of the three possible solutions entirely fulfil the two criteria.

A potential solution to these problems could be a fixed adjustment based on historic data. However, further analysis needs to be carried out in this respect.

EIOPA is interested in stakeholders view on the adjustment for pegged currencies in particular other suggestions to approximate the adjustment notwithstanding the criteria of hedging and liquidity.

4 Extrapolation and interpolation

The appropriate risk-free interest rate term structure will in practice be constructed from a finite number of liquid market data points. Therefore, both interpolation between these data points and extrapolation beyond the last available data point of sufficient liquidity is required.

The input to the interpolation and extrapolation process will consist of the raw market data which is derived based on the approaches outlined in the previous sections. To interpolate and to extrapolate from these data, either the same method can be applied (as done in the Smith-Wilson Method or in the Nelson-Siegel Method), or different methods can be used; e.g. cubic splines or bootstrapping for the interpolated part, and Nelson-Siegel or any other method for the extrapolated part.

Irrespective of whether the same or different methods are chosen for the different parts of the curve, attention should be paid to the consistency of the interest rate curves. The risk-free interest rate term structure should be determined on the basis of a holistic and consistent approach to the setting of all assumptions and parameters on which the curve is based ensuring consistency over time and avoiding artificial volatility of technical provisions and eligible own funds in excess of the capital requirements. Thus, at each valuation date, the interest rates should be consistent across all maturities; besides, when rolling the term structure forward from one valuation date to the next a certain consistency between neighbouring maturities during time should be preserved. Otherwise there is the risk of introducing artificial volatility.

The overall aim of extrapolating yield curves is to construct a stable and robust extrapolated yield curve which reflects current market conditions and at the same time embodies economic views on how unobservable long term rates are expected to behave.

The extrapolation of the risk-free curve significantly impacts the present value of long term insurance liabilities. Small differences in the extrapolated part of the term structure may lead to huge differences in the amount of technical provisions and thus have broad effects on the balance sheets and results of insurers.

Furthermore, depending on the existence of observable liquid data points, the need for extrapolated rates varies for different currencies. Thus, common principles governing the methods of extrapolation to ensure a level playing field between different currencies need to be established.

One aspect of the extrapolation is the ultimate forward rate (UFR). For each currency, the extrapolated part of the basic risk-free interest rate term structure shall be based on forward rates converging smoothly from one or a set of interest rates in relation to the longest maturities for which the relevant financial instruments and the bonds in that currency can be observed in a deep and liquid market to an ultimate forward rate. This ultimate forward rate has to be assessed in line with long-term economic expectations

The methods that can be used in the extrapolation of interest rates are restricted to methods that comply with the macroeconomic approach.

4.1 Choice of the extrapolation and interpolation methodology

It is considered important that extrapolation - as well as interpolation of the yield curve -also focuses on the main objectives as set out in section 2 to ensure an overall consistent approach to the interest rate term structure.

We will consider all the objectives as set out in section 2 and analyse for the specific macroeconomic method whether it complies with the objective:

1. Continuity: *The interest rate term structure should be theoretically and economically sound, ensuring an arbitrage-free interest rate curve which is continuous.*

   All the methods that so far have been studied as possible methods[17] can be deemed as theoretically and economically sound methods (provided that the parameters are assessed in a theoretically and economically sound way). They result in at least continuous spot rate curves, which - with the exception of the linear method - are also continuously differentiable (i.e. forward rates are continuous).

   Concerning the arbitrage-free property, it can be demonstrated that the Smith-Wilson extrapolation is arbitrage free[18]. On the other side, the Nelson-Siegel method cannot ensure theoretically the absence of arbitrage opportunities[19]. However, academic literature exists showing that in certain circumstances there is no statistically significant difference between Nelson-Siegel and an arbitrage free model[20].

2. Smoothness and robustness: *The methodology to determine the interest rate term structure should provide for smooth interest rate curves avoiding artificial volatility in valuation and provide for a reasonable variation over time.*

   A curve is said to be "smooth" if the function defining the curve has at least a continuous first derivative. Curves that allow a continuous derivation are more preferable, because they seem more natural and they perform better in all kinds of models. Based on this definition, only the linear extrapolation gives interest rate term structures that are not smooth.

   Further on, the question would be what has to be seen as "natural" volatility and what can be deemed as "artificial" volatility when monitoring the development of the interest rate term structure over time (e.g. over a certain valuation period).

   The main argument against producing the more realistic transaction prices (which can be quite volatile and bumpy if the rates in the last part of liquid

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[17] The methods that were vetted besides the Smith-Wilson method, are the Nelson-Siegel method and the Svensson method in a macroeconomic formulation (i.e. the parameters determining the level of the long forward rate curve and the speed of convergence were taken as exogeneous parameters), the method developped and used by Barrie and Hibbert, and the linear approach as used in QIS5.

[18] Oral communication with Andrew Smith at presentation given to EIOPA's Technical Provision subgroup meeting on April 13, 2011 in Frankfurt.

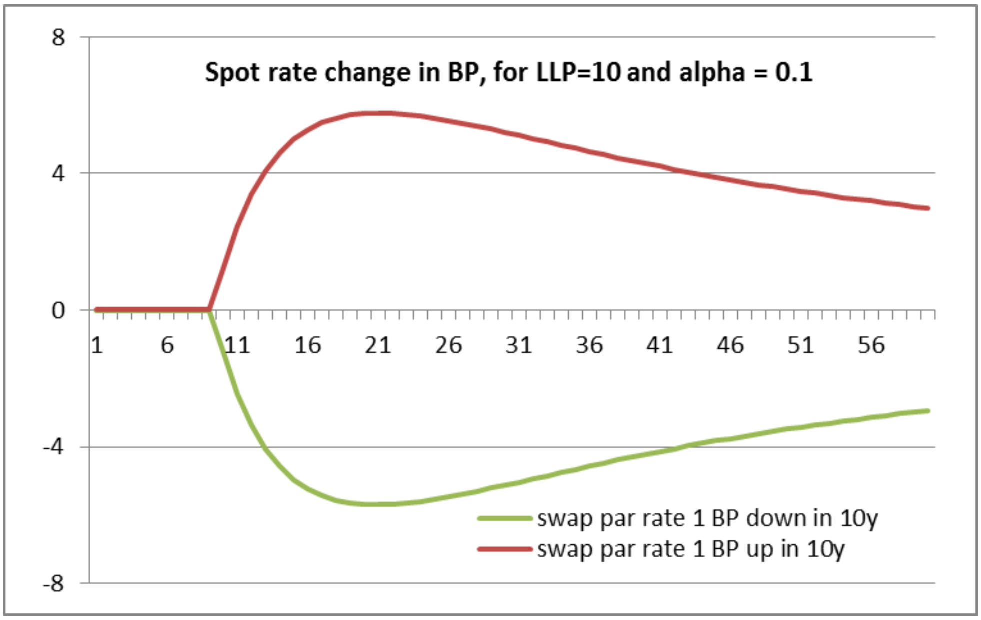
[19] Bjork, T. and B.J. Christensen (1999) "Interest Rate Dynamics and Consistent Forward Rate Curves" Mathematical Finance 9, 323–348

[20] Laura Coroneo, Ken Nyholm and Rositsa Vidova-Koleva: HOW ARBITRAGE-FREE IS THE NELSON-SIEGEL MODEL? The paper can be downloaded from http://www.ecb.europa.eu or from the Social Science Research Network electronic library at http://ssrn.com/abstract_id=1091123.

market data experience large changes) is that in certain economic situations they may have a pro-cyclical effect.

However, there is a second aspect to this question. The extrapolation method can be found to translate small changes in the rates in the last part of the liquid market data into large changes of the extrapolated curve, and thus ultimately into large changes for technical provisions.

Do stakeholders think that "artificial volatility" in the extrapolation is a concern and if so, how should it be addressed (refer also to section 9.3.4. in the Annex)?



Sensitivity in extrapolated spot rates for alpha=0.1, when the year 10 swap par rate is changed by one basis point.

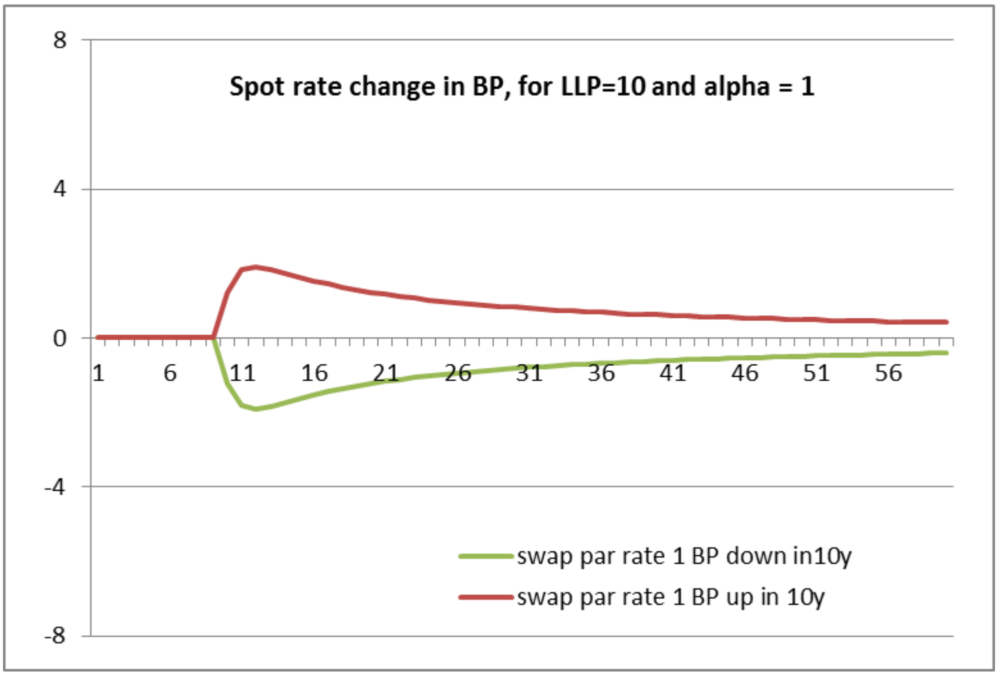


Table 2: Sensitivity in extrapolated spot rates for alpha=1, when the year 10 swap par rate is changed by one basis point.

In the above figures, an illustration is given on how a small change in the last liquid market rate (of plus and minus 1 basis point), influences the extrapolated spot rates at longer tenors in the Smith-Wilson method. The illustration is based on the par swap values for NOK as of 29.06.2010. Further assumptions are that the entry point to the extrapolation is 10 years, the UFR is 4.2%, no Credit risk adjustment, countercyclical premium or Matching premium are included and alpha is in table 1 set at 0.1, and in table 2 at 1.

It can be seen that in general, when the last point on the liquid market data curve flattens or steepens, the effect on longer term tenors can be material in the Smith-Wilson extrapolation. Thus, market participants could buy and sell heavily during the hours they know the assessment of term structure will be based on, in an attempt to influence the Solvency II outcome: These are concerns relevant to all currencies but especially where liquid market data exists only up to 10 years, and where the whole market is deemed to be relatively small. Solutions to this can either be considered in the choice of liquid market data as input to the extrapolation method or in the extrapolation method and its parameters.

3. <u>Accuracy:</u> *The methodology should provide a good fit to the liquid market data*

   Where outliers are systematically identified and eliminated, fitting data points exactly would not result in bumpy spot yield curves. However, bumpy forward yield curves could still occur. This is sometimes tried to be avoided as it is deemed to be not so intuitive, but as a perfect fit to market data has been reached the term structure has to be considered to be market consistent.

While there is no reason to not exactly fit the data in the liquid market part, some more consideration should be given to the transition from interpolated to extrapolated rates. Barrie&Hibbert take a macro-economic view on how this transition should look like. Instead of fitting the extrapolated data in the last 20% of the liquid part to the observed market data, they constrain the estimated data to follow a smooth path that is defined through the targeted UFR and the speed of convergence to the UFR in the extrapolated part of the curve. The resulting curve is smoother (in the sense of less bumpy) and more robust than if fitting exactly to observable market data. But the price it comes at is pricing errors for instruments with maturities in the range where the smoothing was performed.

Taking into account that a lot of expert judgment is needed to perform the smoothing spline procedure as described above, and that it requires the use of more complex optimization methods, the estimation procedure and thus replication by market participants will be both more complicated and more ambiguous as when using e.g. the Smith-Wilson method with an exact fit.

4. Transparency and practicability: The methodology should be fully transparent, easy to apply and accessible to undertakings to ensure applicability in times where EIOPA does not publish the interest rate term structure.

   All the methods that have been analysed, the only exception being the method of Barrie&Hibbert, are open domain methods and fully transparent and fully accessible. For the Smith-Wilson method companies that need the discount rate for unpublished currencies, or at valuation dates when EIOPA does not publish, could easily develop an own excel-spread sheet. In this spread sheet the raw market data would be the input and the term structure interest curve (spot rates for all maturities from 1 to 135 years) would be the output.

   The method of Barrie&Hibbert may have some advantages, but it is ruled out as it is a proprietary method. Undertakings that need a curve that is not published by EIOPA would have to buy a tool and access to expert judgment from Barrie&Hibbert.

5. Incentives: *The methodology should not give inappropriate risk management incentives.*

A trade-off has to be made between smoothness of the curve, accessibility, market consistency and transparency of the methodology. The Smith-Wilson method is seen as the macroeconomic technique that embodies best all of the above objectives. But also the Smith-Wilson method is far from perfect and has some weaknesses[21].

Do you agree that the Smith-Wilson method is the right approach for the purpose of deriving a risk free discount curve for insurance cash flows?

[21] Please refer to the annex for the discussion of advantages and disadvantages of the Smith-Wilson approach.

4.2 Parameterisation of the extrapolation methodology

4.2.1 Setting the ultimate forward rate

The assumption for the extrapolation in Solvency II is that the forward rates at the long end of the term structure converge to a macro-economically assessed ultimate forward rate (UFR). While being subject to regular revision, the ultimate long term forward rate should be stable over time and only change due to changes in long-term expectations.

The most important economic factors explaining long term forward rates are long-term expected inflation and expected real interest rates. From a theoretical point of view it can be argued that there are at least two more components: the expected long-term nominal term premium and the long-term nominal convexity effect.

The term premium represents the additional return an investor may expect on risk-free long dated bonds relative to short dated bonds, as compensation for the longer term investment. This factor can have both a positive and a negative value, as it depends on liquidity considerations and on preferred investor habitats.

The convexity effect arises due to the non-linear (convex) relationship between interest rates and the bond prices used to estimate the interest rates. This is a purely technical effect and always results in a negative component.

No term premium shall be included in the UFR. In order to have a robust and credible estimate for the UFR the assessment is based on the estimates of the expected inflation and the expected short term real rate only.

Making assumptions about expectations this far in the future for each economy is difficult. However, in practice a high degree of convergence in forward rates can be expected when extrapolating at these long-term horizons.

The UFR is reached within 40 years past the last liquid point, in the sense that it is reasonably close (i.e. not more than 3 BP away) to the UFR.

For reasons of pragmatism and the fact that it is impossible to credibly assess nominal interest rates 50 or more years from now, EIOPA suggests to use only one UFR, currently set at 4.2% (i.e. 2.2% long term growth rate and 2% inflation rate assumption).

EIOPA is cognisant of the fact that these are simplifying assumptions and hence invites participants to share their view on this approach. EIOPA is particularly interested of whether industry sees a need for further buckets.

4.2.2 Setting the speed of convergence parameter alpha

In the Smith-Wilson approach, both interpolation between the liquid market data points and extrapolation beyond the last data point are performed simultaneously. This has consequences for the choice of an appropriate alpha. The alpha parameter in the Smith-Wilson approach determines both the speed of convergence to the UFR in the extrapolated part, and the smoothness of the curve in the interpolated part. Larger values of alpha give greater weight to the ultimate forward rate, while smaller values of alpha give more weight to the liquid market data. The choice of alpha has to balance the following two requirements: stability and robustness (would require higher alphas) on the one side and market consistency of the valuation (would require lower alpha) on the other side.

The condition of the UFR being reached at least 40 years beyond the start of the extrapolation is translated into an algorithm to assess the convergence parameter alpha. This approach provides a minimum alpha. However, it has to be considered that other aspects may require a higher alpha.

Algorithm:

i. Set the default value for alpha to 0.1.

ii. Recalibrate alpha in steps of 0.01 upwards as long as the extrapolated rates at point of entrance into extrapolation plus 40 years deviates by more than 3 bps from the UFR.

In addition to this algorithm, the following two technical considerations have to be taken into account when choosing alpha:

- There is no constraint in the method that forces the discount function *P(t)* to decrease. In the liquid part of the assessed term structure we could have cases where *P(t)* is a decreasing function on the given liquid market data points, but becomes a locally increasing function on the interpolated curve (i.e. estimated forward rates of the interpolated part between market data become negative). This can happen if a smooth curve is fitted to two neighbouring market data points for which the P(t) values are quite near. As an example, if *P(0)*=1, *P(1)*=0.95001, *P(2)*=0.95000, and *P(3)*=0.9, and we fit a smooth curve through this points, we will for larger *alpha* get a curve between *P(1)* and *P(2)* that will bend down (i.e. *P(t)*<*P(2)*=0.95 for some 1<*t*<2) because of the enforced smooth continuation of the fit between *P(0)* and *P(1)*. (This is not only a problem with Smith-Wilson, but also with many other methods). Decreasing the alpha will diminish the problem, but only up to a certain degree. Besides, decreasing the alpha will also have influence on the extrapolated part of the term structure. In this case, expert judgment has to be used in order to decide whether the data points can be deemed to be consistent, or whether eventually one of the two market data points that lead to the problem described above can be ignored as an outlier.
- Beyond the liquid part of the curve, *P(t)* may become negative, given that the last forward rate in the liquid part of the curve is high compared to the sum of *UFR* and *alpha* (more details can be found in the annex on extrapolation, under "Lower bound for the parameter alpha in Smith-Wilson"). As an example, for an *alpha* of 10% (i.e. *alpha*=0.1) and the UFR=4.2% the Smith-Wilson method works well when interest rates do not exceed 14,2%. Otherwise, the *alpha* has to be increased. For *an alpha* of e.g. 20% (i.e. *alpha*=0.2) and the UFR=4.2%, the Smith-Wilson method works properly up to interest rates of less than 24.2%. If for certain sets of input market data the alpha that was chosen becomes lower than or equal to the difference of the last forward rate in the liquid part of the curve and the UFR (a value that we will call "lower bound for alpha") the value of *alpha* will be increased in steps of 0.01, until the alpha is higher than the lower bound. This procedure is made rules based in Solvency II.

When the last two points on the liquid market data curve flatten or steepen the effect on longer term tenors can be material. A solution to this problem could be to either take the average of the market price during the last 5 days before valuation date (at least for the two last liquid instruments), or to start the algorithm for all currencies with a higher alpha (alpha 0.5 instead of 0,1)?

5 Counter-cyclical premium

Discussions are still on-going. Hence, this aspect does not form part of the pre-consultation but is mentioned for the sake of completeness

6 Matching Premium

Discussions are still on-going. Hence, this aspect does not form part of the pre-consultation but is mentioned for the sake of completeness.

7 Procedural aspects

The aspects below should be regarded separate from the other sections of the pre-consultation. They will not form part of the implementing technical standards. Nevertheless, EIOPA appreciates any comments as they will help in the decision making process.

Number of currencies published

Having conducted a survey among Member States (MS), EIOPA has decided to derive and publish the risk free term structures for 35 currencies (cf. Annex). This list covers most currencies that were also provided in QIS5 plus a small number of additional currencies that were prioritised by some MS as well. Currencies that are not considered anymore: Estonian kroon (Euro is legal tender since 2011), Hong Kong dollar, Indian rupee, Malaysian ringgit, Thai baht and new Taiwan dollar.

EIOPA is not able to provide curves for every single currency. It is well accepted that the list of 35 currencies is unable to cover all markets European insurance undertakings do business in. But it should cover the largest share of it. The list of currencies may be amended over time if there is a respective need.

Do you agree with the selection that was taken? If no, which currencies would you add?

Frequency of publication:

EIOPA has consulted a number of representatives of the European insurance industry in order to take an informed decision on the frequency of curve publication. The industry generally showed a strong preference for a high frequency of publication, i.e. weekly or daily. EIOPA has to balance this demand against the resource implications this would have for the organisation. As a compromise solution it is hence suggested to group the currencies into three buckets: highly frequent publication, less frequent publication, no publication. The 35 currencies fall into the two first categories. For curves that are provided less often, it was agreed that the absolute minimum is a publication at quarter end.

Please state which currencies of the list of 35 should fall into the first bucket (higher frequency).

Please provide and explain your preferred frequency of publication for each bucket.

Treatment of unpublished curves:

Derivation of risk free term structure: A number of insurance undertakings have obligations in currencies whose risk free term structures will not be published by EIOPA. In general, in order to guarantee consistency, firms have to use the EIOPA curves. If data are not provided by EIOPA, insurance undertakings have to derive for themselves the risk free term structures in accordance with the implementing measures (level 2) and implementing technical standards. The same applies for term structures that are less frequently provided by EIOPA than required by the insurance undertaking.

Use of data providers: EIOPA did some analysis comparing different data providers in order to understand whether the input data required are consistent. For a number of reasons data are not always identical. Nevertheless, the differences were generally seen to be within an acceptable range and therefore sufficiently consistent. Hence, and in order to prevent favouritism, EIOPA does currently not intend to prescribe the use of a specific provider. However, EIOPA will continue to monitor any developments.

Interaction with supervisor: Given the generally moderate business in the unpublished currency markets, it is deemed sufficient for insurance undertakings to derive the risk free term structures according to the Solvency II requirements (level 2 and ITS). It is not assumed that the curves derived by independent experts following the technical standards will be 100% identical. However, the requirements should allow sufficient consistency. Firms do not have to apply for approval by the supervisor every time they derive the risk free term structure. Nevertheless, supervisors reserve the right to request details from the firm and challenge the approach on an ad hoc basis or within the supervisory review process (SRP) if the supervisor considers it is necessary or if the results considerably vary across firms. It is expected that undertakings will document all the relevant data and assumptions used to derive the curve and include those details in their regular supervisory reporting (RSR) on technical provisions

The review will also be used as input to supranational discussions in order to help assure consistent application across jurisdictions.

Internal governance process for derivation of risk free term structure: EIOPA has not finalised its thinking on the internal governance process. This is hence not part of the pre-consultation but comments are welcome.

Treatment of countercyclical premium (CCP) for unpublished currencies: EIOPA will constantly monitor the developments in the financial/capital markets. Necessarily, the main focus will have to be on the 35 chosen currencies both in terms of assessing the general market developments but also to inform the triggering decisions for the CCP. For unpublished currencies, this decision will <u>not</u> be subrogated to insurance undertakings but will also be taken by EIOPA. However, insurance undertakings should alert EIOPA if according to an appropriate representative portfolio the triggering of the CCP is deemed appropriate for this particular currency. It is then for EIOPA to decide whether the CCP will eventually be triggered. Once in place, the processes for the CCP are the same as for any currency[22].

[22] This approach largely hinges on the assumption that insurance business in unpublished currencies is very moderate.

8 Input factors to provided risk free term structures

For the purpose of the pre-consultation, EIOPA provides the risk free curves for three currencies, i.e. EUR, GBP and USD. Below the relevant factors are provided.

Please bear in mind that, as indicated in this document, decisions have not been reached on every single aspect. Thus, some input factors may be reconsidered during further assessments.

Currencies	EUR, GBP, USD
Calculation date	31/12/2010 31/12/2011
Data used	Swap bid rates (government bonds not applicable for selected currencies)
Credit risk adjustment	10bp* across maturities
Entry point of extrapolation	EUR: 20yrs GBP: 50yrs* USD: 30yrs*
Method applied to extrapolation and interpolation	Smith-Wilson approach* Smith-Wilson approach
Speed of convergence factor alpha	0.1*
Maximum period of convergence	40y years from entry point to extrapolation
Ultimate forward rate	4.2%

* same approach as in QIS5

9 Annex

9.1 Assessment measures for ADLT criteria (under development)

Criterion	Definition (TP21 para 3)	Possible Quantitative Measures	Measure feasible for govt bonds or swaps?	Any Qualitative Measures?
Active	An active market is a market where all the following conditions exist: • The items traded within the market are homogenous; • Willing buyers and sellers can normally be found *at any time*; and • Prices are available to the public. or An active market is a market in which transactions for the asset or liability take place with sufficient frequency and volume to provide pricing information *on an ongoing basis*.	*Homogeneity* – taken as read that for a given maturity, vanilla swap and government bond markets consist of homogeneous instruments. *Willing buyers and sellers found at any time* – will be covered by assessment of liquidity criterion (and possibly also indirectly by depth criterion) *Prices available to the public* – will be covered by transparency criterion *Pricing information available on an ongoing basis* – this is linked to transparency and trade frequency. Suggest we seek a measure of trade frequency, possibly also for the purposes of assessing the liquidity criterion.	Unclear whether a suitable measure of (or proxy for) trade frequency exists in public domain since instruments trade in OTC markets. Use of a proxy measure could be problematic if not very transparent and not based on public data only. Liquidity criterion partly covers this aspect as well	
Deep	Transactions involving a *large quantity* of financial	Amihud's ILLIQ measure (uses end-day prices and daily trade volumes – may be	Both ILLIQ and other 'large trade/large price	

	instruments used in the replications can take place without *significantly affecting* the price of the instruments	easier to meet data requirements for this. Possible comparison of 'large' trades (e.g. of some multiple of normal trade size) with 'significant' price moves. Would intra-day information be needed? Probably not available due to OTC markets. Depth can be assessed in terms of amount of liquidity supply/demand at prices just outside the current price range. Very difficult to assess this in opaque OTC markets however. Contributor counts (as in CRO/CFO paper)	swing' comparison methods rely on some measure of trade volumes. Unclear at present whether this is reliably available from public data sources – suspect not.	
Liquid	Financial instruments can *readily* be converted through an act of buying or selling without causing a *significant movement* in the price	Bid-ask spread Typical measure used as a proxy for 'ease' of trading. Tight spreads usually indicative of large number of market participants and also of reasonable depth, but bid-ask spread may need to be supplemented by other measures. Trade frequency – linked to 'readily converted' condition. High trade frequency suggests wouldn't have to wait long/pay a premium to trade 'now'. However, unclear if a good measure of trade frequen-	Bid-ask spread data is available from Bloomberg. See above comments on trade frequency and below comments on regularity of price and trade information.	

		cy exists for government bonds/swaps.		
Transparent	*Current* trade and price information is *readily available* to the public, in particular to the undertakings	A binary measure; '1' if information is available, '0' if not? A scale based on amount/quality/frequency of published trade and price information: '1' for very limited info, to '5' for immediate and good quality full info on trades and prices? [Neither is truly 'quantitative' in sense that assigning numerical scale to subjective judgement. Could look at 'true' quant measures such as time since last trade price. Or could just do a one-off assessment/survey for each currency & market of what data is available from Bloomberg etc. and repeat at some infrequent interval.]	Yes, but full automation or removal of subjectivity from the assessment likely to be challenging. Since transparency should be fairly static condition, suggest a one-off or infrequent assessment interval.	Assessment method likely to be combination of automated/judgement based approach.

		Need for 'current' trade/price information suggests that stale price info could be an indicator of *il*liquidity.		

9.2 Assessment of LLP^{bonds} and entry point to extrapolation

As highlighted in section 3.1, in order to derive the entry point to extrapolation for any currency, the last liquid points for the reference instruments and the bonds have to be determined according to the formula $LLP = \min(LLP^{reference}, LLP^{bonds})$. While there is agreement on the determination of $LLP^{reference}$, views varied on the analysis of LLP^{bonds}. The question is whether the assessment of LLP^{bonds} consists of one or two separate steps. Note that the order of discussion does not constitute a preference by EIOPA.

1. 1 step approach (stage 3 in the overall assessment)

Bond market assessment:

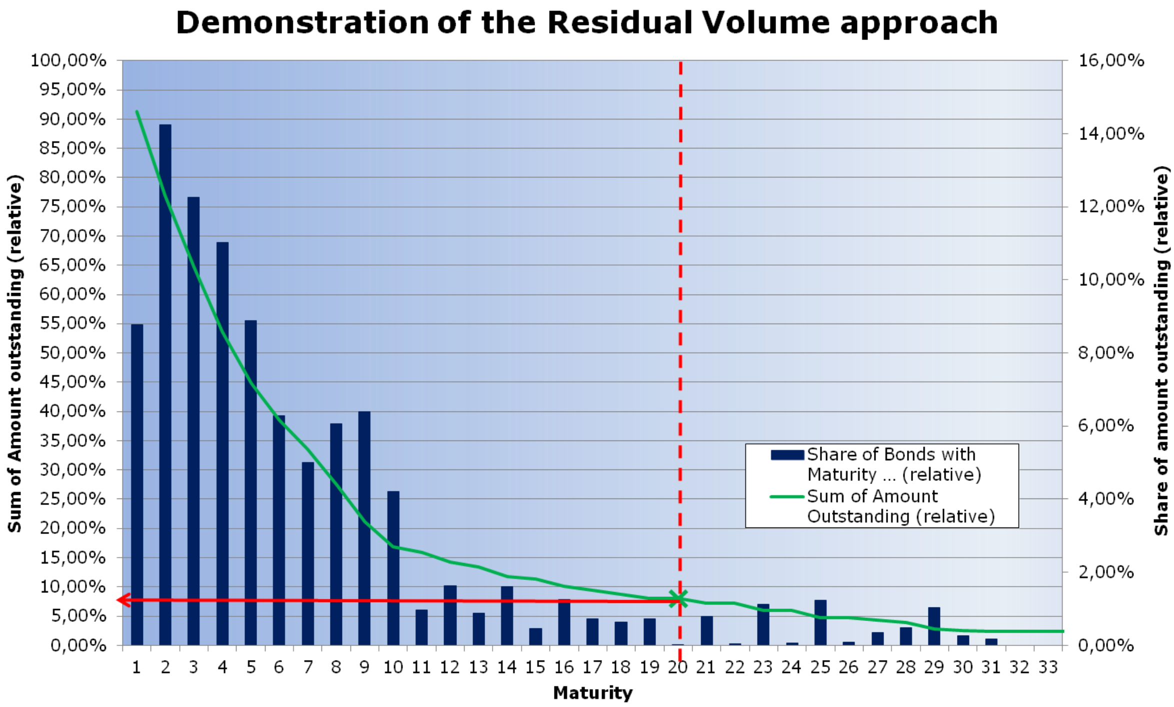
The general criteria of deep, liquid and transparent markets as outlined in section 3.1.3) would apply to both the market for reference instruments and for the bond markets.

However, in respect of the bond markets, it would seem helpful to introduce additional and more specific criteria to support a consistent and robust application of the requirements.

Using the example of the euro, below we will outline one such potential criterion, called the Residual Volume Approach (RVA). Although this is illustrated only for the Euro, this could easily be extended to other currencies.

The RVA criteria is intended to support the assessment of the maturity at which the market for bonds ceases to be deep, liquid and transparent and beyond which undertakings are not able to match their cash-flows with bonds anymore.

The RVA criteria is based on the introduction of a pre-specified threshold of x% of residual bond volume (= percentage of the overall amount outstanding from the respective maturity onwards). The bond market will be considered as deep, liquid and transparent as long as the residual bond volume exceeds this threshold. Thus the last liquid point for bonds would then be located where the residual bond volume is close to x%. The residual bond volume could be deduced from the "IBoxx EUR Overall" index. The index is a good proxy for the total available amount of fixed income markets denominated in euro. The diagram below demonstrates how this approach works:



IBoxx EUR Overall

The chart indicates that if one pre-defines the threshold to be 6%, the market for bonds ceases to be deep and liquid around year 20.

The pre-defined threshold of 6% can now be used to transfer the approach to a wide range of currencies or different market situations, finding the LLP by means of a suitable index. If, for example, the volume of the bond market for the euro increased for long maturities, the residual volume approach would shift the 6%-threshold of residual bond volume to a later maturity.

The data used in the diagram for the € is given below.

Maturity	Volume [€]	Outstanding Amount
1	609.149.052.172	91,05%
2	988.668.515.653	76,52%
3	850.280.448.041	64,02%
4	764.497.965.012	52,78%
5	616.879.208.198	43,72%
6	435.866.805.800	37,31%
7	347.618.928.000	32,20%
8	421.245.023.894	26,01%
9	442.933.046.000	19,50%
10	292.392.087.000	15,20%
11	67.700.684.640	14,21%
12	113.119.449.009	12,54%

Maturity	Volume	Outstanding Amount
26	5.650.000.000	2,73%
27	23.889.000.000	2,38%
28	33.300.728.000	1,89%
29	72.085.735.000	0,83%
30	18.037.060.000	0,57%
31	12.000.000.000	0,39%
32	0	0,39%
33	0	0,39%
34	0	0,39%
35	600.000.000	0,38%
36	1.400.000.000	0,36%
37	0	0,36%

13	61.635.099.000	11,64%	38	0	0,36%
14	111.527.616.118	10,00%	39	0	0,36%
15	31.782.097.875	9,53%	40	0	0,36%
16	87.141.984.515	8,25%	41	0	0,36%
17	49.572.460.458	7,52%	42	0	0,36%
18	43.635.379.209	6,88%	43	16.596.000.000	0,12%
19	50.567.750.000	6,14%	44	0	0,12%
20	1.449.694.549	6,12%	45	0	0,12%
21	54.196.952.600	5,32%	46	0	0,12%
22	3.050.000.000	5,27%	47	0	0,12%
23	77.581.526.800	4,13%	48	8.167.000.000	0,00%
24	4.435.347.000	4,07%	49	0	0,00%
25	85.119.154.000	2,82%	50	0	0,00%

To ensure a reliable valuation of liabilities through time, the choice of the starting point into extrapolation should have due regard to the volume of the available bond markets (i.e. government bonds and corporate bonds[23]) with respect to the liabilities to be covered for the respective maturities, thus taking into account the hedging possibilities of the insurer. Where there is no longer a reliable market for relevant investments for longer maturities, extrapolation should start, to reflect that market data will become more unreliable in the course of time.

Such an assessment of the degree to which insurers are able to match their liabilities with bonds should be carried out with the aim to support the determination of the LLP for bond markets in step 3 and may hence supplement the application of the criteria as outlined in sections 3.2.2 and the assessment outlined above. For example, the results of such an assessment may be used to support the calibration of the pre-specified parameter used in the residual volume approach in context of an ADLT assessment of bond markets.

One potential methodology to carry out a matching assessment would be the liquidity ratio approach, as presented by the CEA[24].

Entry point to extrapolation:

Since the nature of the insurance liabilities and the bond markets in a given currency are both fairly static quantities (i.e. they will change slowly over time if at all), the assessment in the three [under alternative option: four] stages outlined above would not need to be repeated at frequent intervals. The situation should be monitored with reassessments on a yearly basis. The assessment should also be re-performed on an ad hoc basis if there was evidence to suggest that a material change had occurred.

[23] Corporate bonds used as synonym for bonds that are not sovereigns, hence also cover financials.

[24] It is doubtful though whether the liquidity ratio approach in its proper form can be applied in practice due to the lack of data.

Illustrative example of the 3 steps of the assessment of the liquid part of the risk free term structure:

Availability/transparency	**ADLT assessment**	
Stage 1:	**Stage 2:**	**Stage 3:**
Longest maturity for which swaps or government bonds are available and markets for these instruments are transparent	Longest maturity for which swaps or government bonds (i.e. the reference instruments) meet ADLT conditions	Longest maturity for which bonds (i.e. government plus corporate bonds)_ meet ADLT conditions
28 years	25 years	20 years

In the example above, the entry point to extrapolation would be 20 years.

2. 2 step approach (stages 3 and 4 in overall assessment)

Bond market assessment:

As before, the general criteria of deep, liquid and transparent markets (as outlined in section 3.1.3) would apply to both the market for reference instruments and for the overall bond markets.

As a result, it is envisaged that the same assessment method has to be undertaken on whether the markets are ADLT (cf. 3.1.1 and 3.1.2).

This ADLT assessment, using the same methodology as for the reference instruments constitutes stage 3.

To derive the LLP for bonds (LLP^{bonds}) further requirements have to be fulfilled:

To ensure a reliable valuation of liabilities through time, the choice of the starting point into extrapolation should have due regard to the volume of the available fixed income markets (i.e. government bonds and corporate bonds[25]) with respect to the liabilities to be covered for the respective maturities, thus taking into account the hedging possibilities of the insurer. Where there is no longer a reliable market for relevant investments for longer maturities, extrapolation should start, to reflect that market data will become more unreliable in the course of time.

The starting points of the extrapolation of risk-free interest rates should allow undertakings to match with bonds the cash-flows which are discounted with non-extrapolated rates in the calculation of the best estimate. Furthermore, the bond markets are considered active, deep, liquid and transparent, where they allow undertakings to match their insurance cash-flows.

This criterion implies that the total size of insurance liabilities in the market needs to be compared to the cash flows generated by available bond investments in the market.

This is step 4: Assessment of longest maturity for which insurance liabilities can be matched with bonds.

[25] Corporate bonds used as synonym for bonds that are not sovereigns, hence also cover financials.

Because such an assessment requires knowledge of the profile of insurance liabilities in each particular currency, it is for the national supervisors – with EIOPA performing a coordinating role in this process – to make this assessment. National supervisors[26] together with EIOPA will have to demonstrate the location of the point beyond which liabilities could not be reliably matched. This would then form the entry point for extrapolation.

Potential methods for such an assessment are, for instance, the liquidity ratio approach as suggested by the CEA or a comparison between cash flows of insurance liabilities and bonds.

Step 3 and 4 together define LLP^{bonds}, which can also be summarised as LLP^{bonds} = min(ADLT assessment, matching assessment). This assessment combines an assessment of the maturity up to where bonds fulfil the ADLT criteria (step 3) and where undertakings are able to match cash flows with bonds (stage 4).

The entry point to extrapolation will eventually be derived as the minimum of step 2 ($LLP^{reference}$) and step 3 and 4 (LLP^{bonds})

Entry point to extrapolation:

Since the nature of the insurance liabilities and the bond markets in a given currency are both fairly static quantities (i.e. they will change slowly over time if at all), this assessment would not need to be repeated at frequent intervals. The situation should be monitored with reassessments on a yearly basis. The assessment should also be re-performed on an ad hoc basis if there was evidence to suggest that a material change had occurred.

Illustrative example of the 4 steps of the assessment of the liquid part of the risk free term structure:

Availability/transparency	**LLP/entry point to extrapolation assessment**		
	$LLP^{reference}$	**LLP^{bonds}**	
			Bond matching
Stage 1: Longest maturity for which swaps or government bonds are available	**Stage 2:** Longest maturity for which swaps or government bonds (i.e. the reference instruments) meet ADLT conditions	**Stage 3:** Longest maturity for which bonds (i.e. government plus corporate bonds_ meet ADLT conditions	**Stage 4:** Longest maturity for which insurance liabilities can be matched with bonds
28 years	25 years	23 years	20 years

In the example above, the entry point to extrapolation would be 20 years.

[26] In case of the euro, a coordination between all the respective national regulators and EIOPA will be necessary in the future. At the date of adoption of Solvency II, one may assume that this point is reached at 20 years.

Further illustrative examples:

LLP/entry point to extrapolation assessment					
Reference instruments		$LLP^{reference}$	LLP^{bonds} F= min(D,E)		Entry point to extrapolation
Swaps A	Government bonds B	C= max(A,B)	All bonds D	Matching Assessment E	min($LLP^{reference}$, LLP^{bonds}) = min(C,F)
30yrs	20yrs	30yrs	20yrs	20 yrs	20yrs
30yrs	10yrs	30yrs	20yrs	19yrs	19yrs
10yrs	30yrs	30yrs	30yrs	25yrs	25yrs
15yrs	10yrs	15yrs	25yrs	15yrs	15yrs

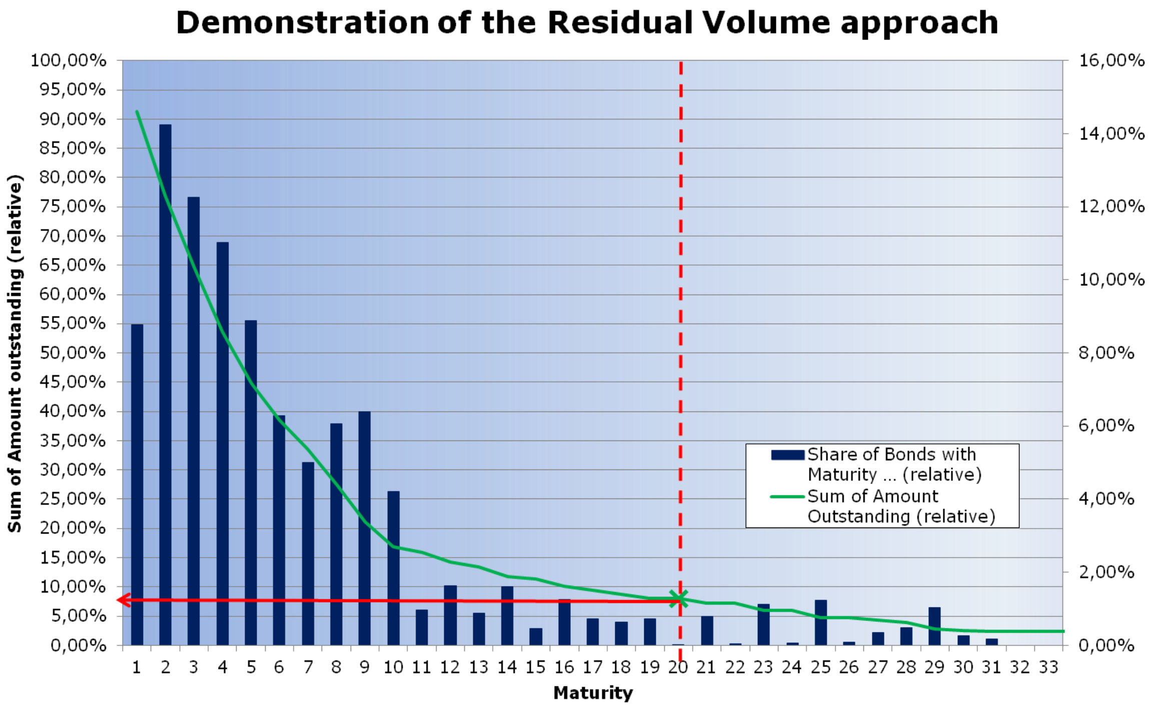
These examples show that the entry point to extrapolation cannot be later than the highest maturity of both reference instruments that still fulfil the ADLT requirements but it can well be earlier, if 1) the bond market does not fulfil the ADLT criteria up to the same maturity or 2) there are not sufficient assets available to match the insurance obligations.

9.3 Assessment of last liquid point for bonds – two examples

Residual Volume Approach

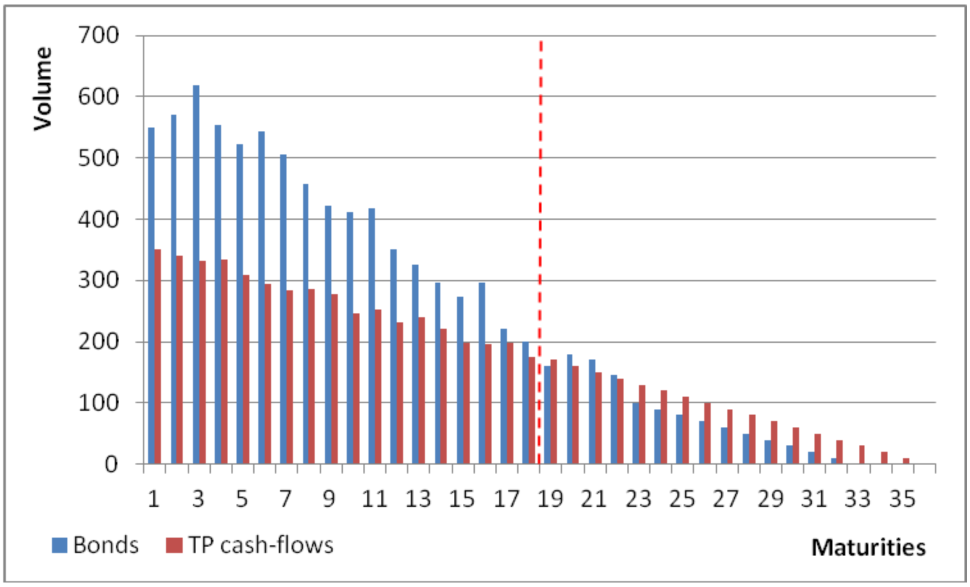
In this section, using the example of the euro, we will illustrate one potential method for determining the last liquid point for bonds. This determination method – called the Residual Volume Approach - can be transferred to any other currency or time.

The idea is to define a maturity threshold where the volume of bonds is not large enough for the market to be considered active, deep liquid and transparent. Details on the approach and how its assessment can be supplemented can be found in the previous section 9.2.



Comparison bond cash flow and cash flows of insurance liabilities

Reference instruments and bonds have to fulfil the ADLT requirements. In addition, the choice of the starting points of the extrapolation of risk-free interest rates should allow undertakings to match with bonds the cash-flows which are discounted with non-extrapolated rates in the calculation of the best estimate. Hence, a comparison between those two cash flows has to be undertaken to ultimately find the LLP^{bonds}. The volume of bonds should be greater or at least equal to the volume of cash-flows resulting from insurance liabilities (undiscounted) in order undertakings are able to match their cash-flows for non-extrapolated part of the basic risk free rate.



Illustrative example

9.4 Extrapolation

9.4.1 The Smith-Wilson method

The Smith-Wilson technique is a macroeconomic approach: a spot (i.e. zero coupon) rate curve is fitted to observed prices of financial instruments, with the macroeconomic ultimate long term forward rate as input parameter.[27]

The approach provides a stable method for extrapolating the yield curve and is consistent with absence of arbitrage. An additional feature of this approach is that it is capable of exactly fitting to the initial term structure, based on a finite set of inputs.

The output from the Smith-Wilson calculation is the discount factor *P(t), t>0*. *P(t)* is the market price at valuing time for a zero coupon bond paying 1 at some future date *t* (the maturity).

Depending on whether we need the spot rates as continuously compounded rates $\tilde{R}_t$ or as rates R_t with annual compounding, the following relation between the discount factor and the spot rate can be used to assess the spot rates: $P(t) = \exp(-t \cdot \tilde{R}_t)$ for continuously compounded rates, and $P(t) = (1 + R_t)^{-t}$ for annual compounding.

The relation between the two rates is $\tilde{R}_t = \ln(1 + R_t)$.

The aim is to assess the price function *P(t)* for <u>all maturities</u> *t, t > 0.* From the relations referred to above it can be seen that therewith the whole risk-free term structure at valuing date is defined.

In its most general form the input data for the Smith-Wilson method can consist of different financial instruments that relate to interest rates. We will first present the formulae in the case where the inputs are zero coupon bond prices. The formulae in this simple case are quite easy to understand and straightforward to implement. Then we will present the formulae for the general case, where a large set of arbitrary financial instruments can be the input.

All financial instruments specified through

- their market price at valuation date,
- the cash payment dates up to maturity, and
- the size of the cash flows at these dates,

can be input instruments for the Smith-Wilson method.

In the Annex 6 we will look at the input for fitting to zero coupon bond rates, to coupon bond rates and to par swap rates, and will present some worked examples.

We will now explain how the term structure can be assessed by using the S-W technique.

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[27] The mathematical background and a further discussion of the method can be found in the original paper by Andrew Smith and Tim Wilson, see Smith A. & Wilson, T. – "Fitting Yield curves with long Term Constraints" (2001), Research Notes, Bacon and Woodrow.

Smith-Wilson for zero coupon bond prices as input

We start by assuming that in the liquid part of the term structure the price function is known for a fixed number of *N* maturities: u_1, u_2, u_3, up to u_N. This is the same as saying that the risk-free zero coupon rates for these *N* liquid maturities are given beforehand.

Depending on whether the market data spot rates are given as continuously compounded rates $\tilde{R}_{u_i}$ or as rates R_{u_i} with annual compounding, the input zero bond prices at maturities u_j can be expressed as:

$m_i = P(u_i) = \exp(-u_i \cdot \tilde{R}_{u_i})$ for continuously compounded rates, and

$m_i = P(u_i) = (1 + R_{u_i})^{-u_i}$ for annual compounding.

In this case, where zero coupon bond prices are the input, the task consists in assessing the price function, i.e. the spot rates for the remaining maturities. These can be both maturities in the liquid end of the term structure where risk-free zero coupon rates are missing (interpolation) and maturities beyond the last observable maturity (extrapolation).

The pricing function proposed by Smith and Wilson[28] reduces in this simple case to:

$$P(t) = e^{-UFR \cdot t} + \sum_{j=1}^{N} \zeta_j \cdot W(t, u_j), \quad t \geq 0 \tag{1}$$

With the symmetric Wilson $W(t, u_j)$ functions defined as:

$$W(t, u_j) = e^{-UFR \cdot (t + u_j)} \cdot \left\{ \alpha \cdot \min(t, u_j) - 0.5 \cdot e^{-\alpha \cdot \max(t, u_j)} \cdot \left(e^{\alpha \cdot \min(t, u_j)} - e^{-\alpha \cdot \min(t, u_j)} \right) \right\} \tag{2}$$

The following notation holds:

- *N,* the number of zero coupon bonds with known price function
- m_i, *i=1, 2, … N*, the market prices of the zero coupon bonds
- u_i, *i=1, 2, … N*, the maturities of the zero coupon bonds with known prices
- *t,* the term to maturity in the price function
- *UFR*, the ultimate unconditional forward rate, continuously compounded
- *a,* mean reversion, a measure for the speed of convergence to the UFR
- ζ_i, *i=1, 2, … N*, parameters to fit the actual yield curve

[28] Smith A. & Wilson, T. – "Fitting Yield curves with long Term Constraints" (2001), Research Notes, Bacon and Woodrow. Referred to in Michael Thomas, Eben Maré: "Long Term Forecasting and Hedging of the South African Yield Curve", Presentation at the 2007 Convention of the Actuarial Society of South Africa.
Andrew Smith: Pricing Beyond the Curve – derivatives and the Long Term (2001), presentation to be found at http://www.cfr.statslab.cam.ac.uk/events/content/20001/asmith2001.pdf

The so called kernel functions $K_j(t)$ are defined as functions of the maturity t:

$$K_j(t) = W(t, u_j),\ \mathrm{t} > 0 \quad \text{and } j=1,2,3,\ldots,N \tag{3}$$

They depend only on the input parameters and on data from the input zero coupon bonds. For each input bond a particular kernel function is computed from this definition. The intuition behind the model is to assess the function *P(t)* as the linear combination of all the kernel functions. This is similar to the Nelson-Siegel method, where the forward rate function is assessed as the sum of a flat curve, a sloped curve and a humped curve, and the Svensson method, where a second humped curve is added to the three curves from Nelson-Siegel.

The unknown parameters needed to compute the linear combination of the kernel functions, ζ_j, $j=1, 2, 3 \ldots N$, are given as solutions of the following linear system of equations:

$$m_1 = P(u_1) = e^{-UFR\cdot u_1} + \sum_{j=1}^{N} \zeta_j \cdot W(u_1, u_j)$$

$$m_2 = P(u_2) = e^{-UFR\cdot u_2} + \sum_{j=1}^{N} \zeta_j \cdot W(u_2, u_j)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{4}$$

$$m_N = P(u_N) = e^{-UFR\cdot u_N} + \sum_{j=1}^{N} \zeta_j \cdot W(u_N, u_j)$$

In vector space notation this becomes:

$$\mathbf{m} = \mathbf{p} = \boldsymbol{\mu} + \mathbf{W}\boldsymbol{\zeta}\ , \tag{5}$$

with:

$$\mathbf{m} = (m_1, m_2, \ldots\ldots., m_N)^T,$$

$$\mathbf{p} = (P(u_1), P(u_2), \ldots\ldots P(u_N))^T,$$

$$\boldsymbol{\mu} = (e^{-UFR\cdot u_1}, e^{-UFR\cdot u_2}, \ldots. e^{-UFR\cdot u_N})^T,$$

$$\zeta = (\zeta_1, \zeta_2, \ldots \zeta_N)^{\mathrm{T}},$$

where the superscript *T* denotes the transposed vector and

$\mathbf{W} = (W(u_i, u_j))_{i=1,2,...N;\, j=1,2,...N}$ is a *NxN*-matrix of certain Wilson functions.

It follows from (5) that the solution $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_N)^T$ is the product of the inverted *NxN*-matrix **W** and the difference between the **p**-vector and the **μ**-vector (i.e. the difference between the market prices of the zero coupon bonds and the asymptotical term), that is:

$$\zeta = \mathbf{W}^{-1}(\mathbf{p} - \boldsymbol{\mu}) = \mathbf{W}^{-1}(\mathbf{m} - \boldsymbol{\mu}), \qquad (6)$$

We can now plug these parameters (i.e. $\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_N$) into the pricing function and get the value of the zero coupon bond price for all maturities t for which no zero coupon bonds prices are given to begin with:

$$P(t) = e^{-UFR \cdot t} + \sum_{j=1}^{N} \zeta_j \cdot W(t, u_j), \quad t>0 \qquad (7)$$

From this value it is straightforward to calculate the spot rates by using the definition of the zero coupon bond price. The spot rates are calculated as $\tilde{R}_t = \frac{1}{t} \cdot \ln(\frac{1}{P(t)})$ for continuous compounded rates and $R_t = (\frac{1}{P(t)})^{\frac{1}{t}} - 1$ if annual compounding is used.

Smith-Wilson technique for a set of general inputs

We now assume that we have N interest related financial instruments as input from the liquid part of the term structure and that J is the number of different dates at which a cash payment has to be made on behalf of at least one of these instruments. The following input shall be given:

- The market prices m_i of the instruments i at valuation date, for $i=1,2,3,\ldots,N$.
- All cash payment dates $u_1, u_2, u_3, \ldots, u_J$, for the instruments.
- The cash flows $c_{i,1}, c_{i,2}, c_{i,3}, \ldots, c_{iJ}$ that are due for instrument i at time $u_1, u_2, \ldots u_J$, for all i. (If no cash payment is due at time $t = u_j$ on instrument i, then $c_{i,j}$ is set to nil).

The general pricing function at valuing time proposed by Smith and Wilson[29] is:

$$P(t) = e^{-UFR \cdot t} + \sum_{i=1}^{N} \zeta_i \cdot (\sum_{j=1}^{J} c_{i,j} \cdot W(t, u_j)), \quad t \geq 0 \qquad (8)$$

with the symmetric Wilson-functions $W(t, u_j)$ defined as in (2) above and the same notation for t, *UFR*, α and ζ_i as was given for the zero coupon case.

[29] Smith A. & Wilson, T. – "Fitting Yield curves with long Term Constraints" (2001), Research Notes, Bacon and Woodrow. Referred to in Michael Thomas, Eben Maré: "Long Term Forecasting and Hedging of the South African Yield Curve", Presentation at the 2007 Convention of the Actuarial Society of South Africa.

The function defined by the inner parenthesis in (8) is called the kernel functions $K_i(t)$:

$$K_i(t) = \sum_{j=1}^{J} c_{i,j} \cdot W(t, u_j), \ \mathrm{t} > 0 \, , \mathrm{i} = 1,2,3,\ldots\mathrm{N} \qquad (9)$$

For each input instrument a particular kernel function is computed. The intuition here is to assess the function $P(t)$ as the linear combination of all the kernel functions.

In the simple case, where the zero coupon prices $P(u_i)$ for certain maturities are given as market price input m_i, i.e. where m_i equals $P(u_i)$ for $i=1,2,3,\ldots,N$, the left side of the linear system of equations (LSE) in (1) is known and it was straight-forward to compute the ζ_i from this LSE. In the general case we have the market prices m_i of the instruments, but the zero coupon prices $P(u_i)$ are not known.

We do know how to assess the market price of an instrument i if all cash payment dates $u_1, u_2, u_3, \ldots, u_J$ for the instrument, the cash flows $c_{i,1}, c_{i,2}, c_{i,3}, \ldots, c_{iJ}$ at times $u_1, u_2, \ldots u_J$, and the discount factors $P(u_j)$, $j=1,2,3,\ldots,J$, are known. Then we have to discount the cash flows $c_{i,j}$ to the valuation date (i.e. multiply $c_{i,j}$ with $P(u_j)$ and sum over all cash flow dates).

$$m_i = \sum_{j=1}^{J} c_{i,j} \cdot P(u_j), \ \mathrm{i} = 1,2,3, \ \ldots\mathrm{.N} \qquad (10)$$

In the above relation, we know the market prices m_i and the cash flows $c_{i,j}$.

We set the definition of the price function for $P(u_j)$ (8) into relation (10) and get the LSE:

$$\begin{aligned}
m_1 &= \sum_{j=1}^{J} c_{1,j} \cdot P(u_j) = \sum_{j=1}^{J} c_{1,j} \cdot (e^{-UFR \cdot u_j} + \sum_{l=1}^{N} \zeta_l \cdot \sum_{k=1}^{J} c_{l,k} \cdot W(u_j, u_k)) \\
m_2 &= \sum_{j=1}^{J} c_{2,j} \cdot P(u_j) = \sum_{j=1}^{J} c_{2,j} \cdot (e^{-UFR \cdot u_j} + \sum_{l=1}^{N} \zeta_l \cdot \sum_{k=1}^{J} c_{l,k} \cdot W(u_j, u_k)) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
m_N &= \sum_{j=1}^{J} c_{N,j} \cdot P(u_j) = \sum_{j=1}^{J} c_{N,j} \cdot (e^{-UFR \cdot u_j} + \sum_{l=1}^{N} \zeta_l \cdot \sum_{k=1}^{J} c_{l,k} \cdot W(u_j, u_k))
\end{aligned} \qquad (11)$$

We can rearrange the above expressions to get:

$$\begin{aligned}
\sum_{j=1}^{J} c_{1,j} \cdot P(u_j) &= \sum_{j=1}^{J} c_{1,j} \cdot e^{-UFR \cdot u_j} + \sum_{l=1}^{N} (\sum_{k=1}^{J} (\sum_{j=1}^{J} c_{1,j} \cdot W(u_j, u_k)) \cdot c_{l,k}) \cdot \zeta_l \\
\sum_{j=1}^{J} c_{2,j} \cdot P(u_j) &= \sum_{j=1}^{J} c_{2,j} \cdot e^{-UFR \cdot u_j} + \sum_{l=1}^{N} (\sum_{k=1}^{J} (\sum_{j=1}^{J} c_{2,j} \cdot W(u_j, u_k)) \cdot c_{l,k}) \cdot \zeta_l \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\sum_{j=1}^{J} c_{N,j} \cdot P(u_j) &= \sum_{j=1}^{J} c_{N,j} \cdot e^{-UFR \cdot u_j} + \sum_{l=1}^{N} (\sum_{k=1}^{J} (\sum_{j=1}^{J} c_{N,j} \cdot W(u_j, u_k)) \cdot c_{l,k}) \cdot \zeta_l
\end{aligned} \qquad (12)$$

In vector space notation we write the left side of (11) as:

$$\mathbf{m} = \mathbf{Cp}, \tag{13}$$

and (12) as:

$$\mathbf{Cp} = \mathbf{C\mu} + (\mathbf{CWC}^{\mathrm{T}})\boldsymbol{\zeta}, \tag{14}$$

with:

$$\mathbf{m} = (m_1, m_2, ...m_N)^T,$$
$$\mathbf{p} = (P(u_1), P(u_2), ...P(u_J))^T,$$

$$\mathbf{C} = \begin{bmatrix} c_{1,1} & c_{1,2} & c_{1,3} & \cdots & \cdots & c_{1,j} & \cdots & c_{1,J} \\ c_{2,1} & c_{2,2} & c_{2,3} & \cdots & \cdots & c_{2,j} & \cdots & c_{2,J} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{i,1} & c_{i,2} & c_{i,3} & \cdots & \cdots & c_{i,j} & \cdots & c_{i,1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{N,1} & c_{N,2} & \cdots & \cdots & \cdots & c_{N,j} & \cdots & c_{N,J} \end{bmatrix}. \qquad \text{the NxJ cashflow matrix}$$

$$\boldsymbol{\mu} = (e^{-UFR \cdot u_1}, e^{-UFR \cdot u_2}, ...e^{-UFR \cdot u_J})^T,$$

$$\boldsymbol{\zeta} = (\zeta_1, \zeta_2, ...\zeta_N)^{\mathrm{T}},$$

and

$$\mathbf{W} = \begin{bmatrix} w(u_1,u_1) & w(u_1,u_2) & \cdots & \cdots & w(u_1,u_i) & \cdots & \cdots & w(u_1,u_J) \\ w(u_2,u_1) & w(u_2,u_1) & \cdots & \cdots & w(u_2,u_i) & \cdots & \cdots & w(u_2,u_J) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ w(u_i,u_1) & w(u_i,u_2) & \cdots & \cdots & w(u_i,u_i) & \cdots & \cdots & w(u_i,u_J) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ w(u_J,u_1) & \cdots & \cdots & \cdots & w(u_J,u_i) & \cdots & \cdots & w(u_J,u_J) \end{bmatrix}$$

the *JxJ* matrix of certain Wilson functions.

Combining (13) and (14) leads to:

$$\mathbf{m} = \mathbf{C\mu} + (\mathbf{CWC}^{\mathbf{T}})\boldsymbol{\zeta} \ , \tag{15}$$

and we see at once that the solution ζ_1, ζ_2, ζ_3, … ζ_N is calculated by inverting the *NxN*-matrix $\mathbf{CWC^T}$ and multiplying it with the difference of the market value vector and the vector assessed as product of matrix **C** with the **μ**-vector, the asymptotical term:

$$\boldsymbol{\zeta} = (\mathbf{CWC}^{\mathbf{T}})^{-1}(\mathbf{m} - \mathbf{C\mu}) \tag{16}$$

Now we can plug these parameters ζ_1, ζ_2, ζ_3, … ζ_N for *t= 1,2,3,…* into the pricing function *P(t)* and get the value of the discount function for all maturities, and thus the term structure for the spot rates.

> ***Remark***: When using swap rates to fit the risk-free term structure, an adjustment to allow for the credit risk in swaps has to be made. Assuming that the adjustment can be expressed as a delta credit risk spread of *Δcr* basis points of swaps above basic risk-free rates, in Solvency IIwill be used

the input data will be adjusted with *Δcr* basis points. This means that *Δcr* basis points are subtracted from the raw par swap rates. These adjusted rates are then used to assess the spot rates with the S-W technique.

Lower bound for the parameter alpha in Smith Wilson

The Smith–Wilson pricing function at valuing time for maturity t (see above) is:

$$P(t) = e^{-UFR\cdot t} + \sum_{i=1}^{N} \zeta_i \cdot (\sum_{j=1}^{J} c_{i,j} \cdot W(t, u_j)), \quad t \geq 0$$

In a first step we can show that for large t (beyond the last liquid market data point) the following relation holds, where the values X_0 and X_1 are constants that depend on the valuation date, on UFR, alpha, **ζ**, the cash payments date and the cash flows in the payment dates of the input instruments:

$$P(t) = e^{-UFR\cdot t} \cdot X_0 + e^{-(UFR+\alpha)\cdot t} \cdot X_1 \tag{i}$$

In a second step we consider the formula (i) in terms of forward rates f(t), t≥0. We can show that:

$$f(t) = UFR + \frac{\alpha e^{-\alpha(t-u_J)}\left(f(u_J) - UFR\right)}{\alpha - (1 - e^{-\alpha(t-u_J)})\left(f(u_J) - UFR\right)} \tag{ii}$$

It is easy to deduce from (ii) that the forward rates in the above formula do not have to remain positive and finite if $f(u_J) - UFR > \alpha$.

To see this take for example $f(u_J) = UFR + \alpha + \varepsilon$ with an arbitrary but non-negative ε, and look at the denominator in the expression for the forward rate:

$$f(t) = UFR + \frac{\alpha e^{-\alpha(t-u_J)}(\alpha+\varepsilon)}{\alpha-(1-e^{-\alpha(t-u_J)})(\alpha+\varepsilon)} = UFR + \alpha\frac{(\alpha+\varepsilon)e^{-\alpha(t-u_J)}}{-\varepsilon+(\alpha+\varepsilon)e^{-\alpha(t-u_J)}}$$

Besides, if ε is zero, i.e. $f(u_J) - UFR = \alpha$, than (ii) reduces to $f(t) = UFR + \alpha$. Thus $f(t) \equiv f(u_J)$ for t larger u_J. This means that the forward rates f(t) are constant beyond the last market point data and do not converge to the UFR (given that $f(u_J)$ was not equal to the UFR to begin with).

Therefore, for the Smith-Wilson method to work appropriately along the whole term structure the *α* should be higher than $f(u_J)$-*UFR*.

Define the value of "f(uJ)-UFR" as the lower bound for alpha, the speed of convergence in the extrapolation with Smith-Wilson in Solvency II

Prove of relation (i):
The vector $\zeta^T = (\zeta_1, \zeta_2, \zeta_3 \ldots \zeta_N)$ can be calculated from the data of the input instruments, the $c_{i,j}$ are the cash flows from cash flow matrix **C**, the. *u_j are the cash payment dates.*

We can write the above pricing function as multiplication of vectors, the Wilson-functions *W(t,u_j), j=1,2,…J being the values in the vector* $\mathbf{w}_t = (W(t,u_j))_{j=1,2,\ldots J}$:

$$P(t) = e^{-UFR\cdot t} + \boldsymbol{\zeta}^{\mathbf{T}} \cdot \mathbf{C}\cdot \mathbf{w_t}$$

For large maturities t, i.e. for t > u_J, the definition for the Wilson functions reduces to:

$$W(t,u_j) = e^{-UFR\cdot(t+u_j)}\cdot\left\{\alpha\cdot u_j - 0.5\cdot e^{-\alpha t}\cdot(e^{\alpha\cdot u_j} - e^{-\alpha\cdot u_j})\right\}$$

Plugging in the definition of the *W (t, u_j)* for large t, i.e. t > u_J, into the pricing function leads to the following representation for P(t):

$$P(t) = e^{-UFR\cdot t} + \boldsymbol{\zeta}^{\mathbf{T}}\cdot\mathbf{C}\cdot\begin{pmatrix} e^{-UFR\cdot t}\cdot(e^{-UFR\cdot u_1}\alpha\cdot u_1) - 0.5\cdot e^{-(UFR+\alpha)\cdot t}\cdot e^{-UFR\cdot u_1}(e^{\alpha\cdot u_1} - e^{-\alpha\cdot u_1}) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ e^{-UFR\cdot t}\cdot(e^{-UFR\cdot u_J}\alpha\cdot u_J) - 0.5\cdot e^{-(UFR+\alpha)\cdot t}\cdot e^{-UFR\cdot u_J}(e^{\alpha\cdot u_J} - e^{-\alpha\cdot u_J}) \end{pmatrix}$$

$$= e^{-UFR\cdot t}(1+\boldsymbol{\zeta}^{\mathbf{T}}\cdot\mathbf{C}\cdot\begin{pmatrix} e^{-UFR\cdot u_1}\alpha\cdot u_1 \\ \ldots\ldots\ldots \\ e^{-UFR\cdot u_J}\alpha\cdot u_J\end{pmatrix}) + e^{-(UFR+\alpha)\cdot t}\cdot\boldsymbol{\zeta}^{\mathbf{T}}\cdot\mathbf{C}\cdot\begin{pmatrix} -0.5\cdot e^{-UFR\cdot u_1}(e^{\alpha\cdot u_1} - e^{-\alpha\cdot u_1}) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -0.5\cdot e^{-UFR\cdot u_J}(e^{\alpha\cdot u_J} - e^{-\alpha\cdot u_J})\end{pmatrix}$$

$$= e^{-UFR\cdot t}\cdot X_0 + e^{-(UFR+\alpha)\cdot t}\cdot X_1$$

As mentioned above, the constant values X_0 and X_1 depend on the valuation date, on UFR, alpha, **ζ**, the cash payments date and the cash flows in the payment dates of the input instruments.

Prove of (ii):
The relation between pricing function and instantaneous forward rate is given through:

$$P(t) = e^{-\int_0^t f(s)\cdot ds}, \text{ i.e. } \quad \ln(\mathrm{P(t)}) = -\int_0^t \mathrm{f(s)ds}$$

Thus:

$$f(t) = -\frac{P'(t)}{P(t)} = \frac{UFR\cdot X_0 e^{-UFR\cdot t} + (UFR+\alpha)\cdot X_1 e^{-(UFR+\alpha)\cdot t}}{X_0 e^{-UFR\cdot t} + X_1 e^{-(UFR+\alpha)\cdot t}} = UFR + \frac{\alpha X_1 e^{-\alpha\cdot t}}{X_0 + X_1 e^{-\alpha\cdot t}}$$

Let us suppose that the last observed cash flow date is u_J. Then the forward rate at time u_J is:

$$f(u_J) = UFR + \frac{\alpha X_1 e^{-\alpha\cdot u_J}}{X_0 + X_1 e^{-\alpha\cdot u_J}}$$

After some manipulation, we can see that

$$f(t) = UFR + \frac{\alpha e^{-\alpha(t-u_J)}\left(f(u_J) - UFR\right)}{\alpha - (1 - e^{-\alpha(t-u_J)})\left(f(u_J) - UFR\right)}$$

This gives the formula (ii) by which $f(t)$ converges from $f(U_J)$ to UFR.

9.4.2 Smith-Wilson method - Fitting to bond prices and swap rates

With the Smith-Wilson technique the term structure can be fitted to all the different financial instruments that may be eligible as basis for assessing the risk-free interest rate curve.

Each set of instruments that is taken as input is defined by

- the vector of the market prices (of *N* instruments) at valuation date,
- the vector of the cash payment dates (*J* different dates) up to the last maturity, and
- the *NxJ*-matrix of the cash flows on the instruments in these dates.

We will now look at this input when the spot rate curve is fitted to zero coupon bond rates, to coupon bond rates and to par swap rates. We will furthermore give some simple computed examples for par swap rates as input.

Instruments	Market prices	Cash payment dates	Cash flow matrix
Zero coupon bonds	• Market prices of the *N* input instruments, given as the per cent amount of the notional amount • The market prices of the zero coupon input bonds translate at once into spot rates for input maturities	• The cash payment dates are the maturity dates of the *N* zero coupon input bonds (i.e. $J=N$)	• An *NxN* matrix with entries: - $c_{i,j}=1$ for $i=j$, and - $c_{i,j}=0$ else. • Remark: The **C** matrix reduces to the unity matrix. It can easily be seen that all the complex formulae given in (8, 9, 11, and 16) reduce to the simpler ones in (1, 3, 4, 6).
Coupon bonds	• Market prices of the *N* coupon input bonds, given as the per cent amount of the notional amount of the bond.	• The cash payment dates are, in addition to the maturity dates of the input bonds all coupon dates. • Order these *J* cash payment dates in increasing order, i.e. $u_1 < u_2 < \ldots < u_J$ • Order the bonds depending on their time to maturity, such that	• An NxJ matrix with the following entries (all *i*): - $c_{i,j}=r_c(i)/s,\ j<t(i)$ - $c_{i,t(i)}=1+r_c(i)/s,$ - $c_{i,j}=0,\ j>t(i),$ where $r_c(i)$ is the coupon rate of i^{th} bond, and s is the settlement frequency. • Remark: We propose to take the simple

		if the time of maturity of the i^{th} bond is denominated with $u_{t(i)}$, the following holds: $u_{t(1)} < u_{t(2)} \ldots < u_{t(N)} = u_J$	approach, and to not allow for day count details

Instruments	Market prices	Cash payment dates	Cash flow matrix
Par swap rates	• The market prices of the N par swap input instruments are taken as unit (i.e. 1). • To receive the swap rate, a floating rate has to be earned, that can be swapped against the fixed rate. To earn the variable rate a notional amount has to be invested. At maturity, the notional amount is de-invested.	• The cash payment dates are, in addition to the maturity dates of the swap agreements all swap rate payment dates. • Order these J cash payment dates in increasing order, i.e. $u_1 < u_2 < \ldots < u_J$ • Order the swaps depending on their time to maturity, such that if the time to maturity of the i^{th} swap is denominated with $u_{i,t(i)}$, the following holds: $u_{1,t(1)} < u_{2,t(2)} \ldots < u_{N,t(N)} = u_J$	• An NxJ matrix with the following entries (all i): - $c_{i,j} = r_c(i)/s,\ j<t(i)$ - $c_{i,t(i)} = 1+r_c(i)/s$, - $c_{i,j} = 0,\ j>t(i)$, where $r_c(i)$ is the swap rate of agreement i, and s is the settlement frequency. • Remark: We propose to take the simple approach, and to not allow for day count details.

Worked examples

When fitting the spot rate term structure to the input data from the following examples, we set the long term forward rate to 4.2% (for annual compounding; i.e. ln(1+0.042) = 0.04114 or 4.114% for continuous compounding), and the alpha parameter to 0.1.

Example 1

Market data input for example 1

Par swap rates	Market prices	Cash payment dates	Transposed cash flow matrix CT (i.e. cash flows of instrument i in column i)
Maturities: 1,2,3,5 years s=1 coupon rates: $r_c(1)$ 1% $r_c(2)$ 2% $r_c(3)$ 2.6% $r_c(5)$ 3.4%	i m_i 1 1 2 1 3 1 4 1	• $u_1=1$, $u_2=2$, $u_3=3$, $u_4=4$, $u_5=5$ • $u_{1,t(1)}=1$, $u_{2,t(2)}=2$, $u_{3,t(3)}=3$, $u_{4,t(4)}=5$	

	1	2	3	4
u_1	1.01	0.02	0.026	0.034
u_2	0	1.02	0.026	0.034
u_3	0	0	1.026	0.034
u_4	0	0	0	0.034
u_5	0	0	0	1.034

The steps in the S-W technique:

The 5x5 matrix of Wilson functions is computed straightforward from formula (2):

$$\mathbf{W} = \begin{bmatrix} 0.009 & 0.016 & 0.022 & 0.027 & 0.031 \\ 0.016 & 0.030 & 0.041 & 0.051 & 0.058 \\ 0.022 & 0.041 & 0.058 & 0.072 & 0.083 \\ 0.027 & 0.051 & 0.072 & 0.090 & 0.104 \\ 0.031 & 0.058 & 0.083 & 0.104 & 0.122 \end{bmatrix}$$

If multiplied with **C** from the right and $\mathbf{C^T}$ from the left, the resulting 4x4 matrix is:

$$\mathbf{CWC^T} = \begin{bmatrix} 0.009 & 0.017 & 0.023 & 0.035 \\ 0.017 & 0.032 & 0.045 & 0.067 \\ 0.023 & 0.045 & 0.065 & 0.097 \\ 0.035 & 0.067 & 0.097 & 0.150 \end{bmatrix}$$

The inverse of this matrix $\mathbf{CWC^T}$ is computed as:

$$\left(\mathbf{CWC^T}\right)^{-1} = \begin{bmatrix} 10658.6 & -10190.4 & 3653.0 & -268.0 \\ -10190.4 & 14337.6 & -7987.5 & 1116.2 \\ 3653.0 & -7987.5 & 6252.3 & -1323.2 \\ -268.0 & 1116.2 & -1323.2 & 426.6 \end{bmatrix}$$

We first multiply the cash flows in **C** with the vector **μ** of the asymptotic terms, and then subtract this vector from the vector of the market values:

$$\mathbf{m - C\mu} = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} - \begin{bmatrix}0.969\\0.959\\0.956\\0.965\end{bmatrix} = \begin{bmatrix}0.031\\0.041\\0.044\\0.035\end{bmatrix}$$

Multiply $\mathbf{(CWC^T)^{-1}}$ with **m-μ**. The resulting vector represents the solution of the LSE that was set up in (15):

$$\zeta = \begin{bmatrix}57.79\\-33.5\\111.40\\-5.47\end{bmatrix}$$

To assess the discount function *P(t)* in arbitrary *t, t>0*, the Wilson functions *W(t,uj), j=1,2,3…J* have to be assessed and multiplied with **C**, as defined in (8). We want to compute the discount factor for *t=4*, and calculate therefore $\mathbf{w^T} = (W(4,u_j))_{j=1,2,3,4,5}$

$$\mathbf{w}^T = [0.27 \quad 0.051 \quad 0.072 \quad 0.090 \quad 0.104]$$

This vector multiplied with $\boldsymbol{C^T}$ gives the values of the kernel functions in *t=4*, i.e.:

$$(\mathrm{K_i}(4))_{\mathrm{i}=1,2,3,4} = \mathbf{w^T C^T} = [0.027 \;\; 0.052 \;\; 0.076 \;\; 0.116]$$

From the linear combination of these kernel functions we get:

$$\mathbf{(w^T C^T)}\zeta = 0.037$$

and adding the asymptotical factor $e^{-0.04114\times4} = 0.8483$, the discount function at maturity 4 years has the value *P(4)*= 0.848+0.037=0.885. This gives a spot rate (with annual compounding) of 3.10%.

We can table the Wilson functions for all maturities (years, month, days) for which risk-free spot rates will be needed, perform the above calculation for each maturity, and thus assess the risk-free interest rate term structure.

Example 2

Market data input for example 2

<table>
<tr><th>Par swap rates</th><th>Market prices</th><th>Cash payment dates</th><th>Transposed cash flow matrix C^T (i.e. cash flows of instrument i in column i)</th></tr>
<tr>
<td>Maturities:
1,2,3,5 years

s=1

coupon rates
$r_c(1)$ 1%
$r_c(2)$ 2%
$r_c(3)$ 2.6%
$r_c(5)$ 3.4%</td>
<td>
<table>
<tr><td>i</td><td>m_i</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>1</td></tr>
<tr><td>3</td><td>1</td></tr>
<tr><td>4</td><td>1</td></tr>
</table>
</td>
<td>

- u_1=0.25, u_2=0.5, u_3=0.75, u_4=1, u_5=1.25, ……. u_{19}=4.75, u_{20}=5,
- $u_{1,t(1)}$ =1, $u_{2,t(2)}$ =2, $u_{3,t(3)}$ =3, $u_{4,t(4)}$ =5

</td>
<td>
<table>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>u_1</td><td>0.0025</td><td>0.005</td><td>0.0065</td><td>0.0085</td></tr>
<tr><td>u_2</td><td>0.0025</td><td>0.005</td><td>0.0065</td><td>0.0085</td></tr>
<tr><td>u_3</td><td>0.0025</td><td>0.005</td><td>0.0065</td><td>0.0085</td></tr>
<tr><td>u_4</td><td>1.0025</td><td>0.005</td><td>0.0065</td><td>0.0085</td></tr>
<tr><td>u_5</td><td>0</td><td>0.005</td><td>0.0065</td><td>0.0085</td></tr>
<tr><td>u_7</td><td>0</td><td>0.005</td><td>0.0065</td><td>0.0085</td></tr>
<tr><td>u_8</td><td>0</td><td>1.005</td><td>0.0065</td><td>0.0085</td></tr>
<tr><td>u_9</td><td>0</td><td>0</td><td>0.0065</td><td>0.0085</td></tr>
<tr><td>..</td><td>0</td><td>0</td><td>…</td><td>…</td></tr>
<tr><td>u_{19}</td><td>0</td><td>0</td><td>0</td><td>0.0085</td></tr>
<tr><td>u_{20}</td><td>0</td><td>0</td><td>0</td><td>1.0085</td></tr>
</table>
</td>
</tr>
</table>

The steps in the S-W technique

The 20x20 matrix of Wilson functions is computed straightforward from formula (2):

$$\mathbf{W} = \left[\begin{array}{cccccccccccccccccccc}
0.001 & 0.001 & 0.002 & 0.002 & 0.003 & 0.003 & 0.004 & 0.004 & 0.005 & 0.005 & 0.005 & 0.006 & 0.006 & 0.006 & 0.007 & 0.007 & 0.007 & 0.007 & 0.008 & 0.008 \\
0.001 & 0.002 & 0.003 & 0.004 & 0.005 & 0.006 & 0.007 & 0.008 & 0.009 & 0.010 & 0.011 & 0.011 & 0.012 & 0.013 & 0.013 & 0.014 & 0.014 & 0.015 & 0.015 & 0.016 \\
0.002 & 0.003 & 0.005 & 0.007 & 0.008 & 0.009 & 0.011 & 0.012 & 0.013 & 0.014 & 0.016 & 0.017 & 0.018 & 0.019 & 0.019 & 0.020 & 0.021 & 0.022 & 0.023 & 0.023 \\
0.002 & 0.004 & 0.007 & 0.009 & 0.011 & 0.012 & 0.014 & 0.016 & 0.018 & 0.019 & 0.020 & 0.022 & 0.023 & 0.024 & 0.026 & 0.027 & 0.028 & 0.029 & 0.030 & 0.031 \\
0.003 & 0.005 & 0.008 & 0.011 & 0.013 & 0.015 & 0.017 & 0.020 & 0.022 & 0.023 & 0.025 & 0.027 & 0.029 & 0.030 & 0.032 & 0.033 & 0.034 & 0.036 & 0.037 & 0.038 \\
0.003 & 0.006 & 0.009 & 0.012 & 0.015 & 0.018 & 0.021 & 0.023 & 0.026 & 0.028 & 0.030 & 0.032 & 0.034 & 0.036 & 0.037 & 0.039 & 0.041 & 0.042 & 0.044 & 0.045 \\
0.004 & 0.007 & 0.011 & 0.014 & 0.017 & 0.021 & 0.024 & 0.027 & 0.029 & 0.032 & 0.034 & 0.037 & 0.039 & 0.041 & 0.043 & 0.045 & 0.047 & 0.049 & 0.050 & 0.052 \\
0.004 & 0.008 & 0.012 & 0.016 & 0.020 & 0.023 & 0.027 & 0.030 & 0.033 & 0.036 & 0.039 & 0.041 & 0.044 & 0.046 & 0.049 & 0.051 & 0.053 & 0.055 & 0.057 & 0.058 \\
0.005 & 0.009 & 0.013 & 0.018 & 0.022 & 0.026 & 0.029 & 0.033 & 0.036 & 0.040 & 0.043 & 0.046 & 0.049 & 0.051 & 0.054 & 0.056 & 0.059 & 0.061 & 0.063 & 0.065 \\
0.005 & 0.010 & 0.014 & 0.019 & 0.023 & 0.028 & 0.032 & 0.036 & 0.040 & 0.043 & 0.047 & 0.050 & 0.053 & 0.056 & 0.059 & 0.062 & 0.064 & 0.067 & 0.069 & 0.071 \\
0.005 & 0.011 & 0.016 & 0.020 & 0.025 & 0.030 & 0.034 & 0.039 & 0.043 & 0.047 & 0.051 & 0.054 & 0.058 & 0.061 & 0.064 & 0.067 & 0.070 & 0.072 & 0.075 & 0.077 \\
0.006 & 0.011 & 0.017 & 0.022 & 0.027 & 0.032 & 0.037 & 0.041 & 0.046 & 0.050 & 0.054 & 0.058 & 0.062 & 0.065 & 0.069 & 0.072 & 0.075 & 0.078 & 0.080 & 0.083 \\
0.006 & 0.012 & 0.018 & 0.023 & 0.029 & 0.034 & 0.039 & 0.044 & 0.049 & 0.053 & 0.058 & 0.062 & 0.066 & 0.070 & 0.073 & 0.077 & 0.080 & 0.083 & 0.086 & 0.089 \\
0.006 & 0.013 & 0.019 & 0.024 & 0.030 & 0.036 & 0.041 & 0.046 & 0.051 & 0.056 & 0.061 & 0.065 & 0.070 & 0.074 & 0.078 & 0.081 & 0.085 & 0.088 & 0.091 & 0.094 \\
0.007 & 0.013 & 0.019 & 0.026 & 0.032 & 0.037 & 0.043 & 0.049 & 0.054 & 0.059 & 0.064 & 0.069 & 0.073 & 0.078 & 0.082 & 0.086 & 0.089 & 0.093 & 0.096 & 0.099 \\
0.007 & 0.014 & 0.020 & 0.027 & 0.033 & 0.039 & 0.045 & 0.051 & 0.056 & 0.062 & 0.067 & 0.072 & 0.077 & 0.081 & 0.086 & 0.090 & 0.094 & 0.097 & 0.101 & 0.104 \\
0.007 & 0.014 & 0.021 & 0.028 & 0.034 & 0.041 & 0.047 & 0.053 & 0.059 & 0.064 & 0.070 & 0.075 & 0.080 & 0.085 & 0.089 & 0.094 & 0.098 & 0.102 & 0.105 & 0.109 \\
0.007 & 0.015 & 0.022 & 0.029 & 0.036 & 0.042 & 0.049 & 0.055 & 0.061 & 0.067 & 0.072 & 0.078 & 0.083 & 0.088 & 0.093 & 0.097 & 0.102 & 0.106 & 0.110 & 0.113 \\
0.008 & 0.015 & 0.023 & 0.030 & 0.037 & 0.044 & 0.050 & 0.057 & 0.063 & 0.069 & 0.075 & 0.080 & 0.086 & 0.091 & 0.096 & 0.101 & 0.105 & 0.110 & 0.114 & 0.118 \\
0.008 & 0.016 & 0.023 & 0.031 & 0.038 & 0.045 & 0.052 & 0.058 & 0.065 & 0.071 & 0.077 & 0.083 & 0.089 & 0.094 & 0.099 & 0.104 & 0.109 & 0.113 & 0.118 & 0.122
\end{array}\right]$$

If multiplied with **C** from the right and $\mathbf{C}^\mathbf{T}$ from the left, the resulting 4x4 matrix is:

$$\mathbf{CWC^T} = \begin{bmatrix} 0.009 & 0.016 & 0.023 & 0.034 \\ 0.016 & 0.031 & 0.044 & 0.066 \\ 0.023 & 0.044 & 0.063 & 0.095 \\ 0.034 & 0.066 & 0.095 & 0.147 \end{bmatrix}$$

The inverse of this matrix $\mathbf{CWC^T}$ is computed as:

$$\left(\mathbf{CWC^T}\right)^{-1} = \begin{bmatrix} 10765.8 & -10328.0 & 3708.9 & -272.7 \\ -10328.0 & 14579.0 & -8136.8 & 1139.6 \\ 3708.9 & -8136.8 & 6381.5 & -1353.7 \\ -272.7 & 1139.6 & -1353.7 & 437.4 \end{bmatrix}$$

We first multiply the cash flows in **C** with the vector **μ** of the asymptotic terms, and then subtract the vector from the vector of the market values:

$$\mathbf{m - C\mu} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \mathbf{C} \cdot \begin{bmatrix} 0.9898 \\ 0.9796 \\ 0.9696 \\ 0.9597 \\ 0.9499 \\ 0.9402 \\ 0.9305 \\ 0.9210 \\ 0.9116 \\ 0.9023 \\ 0.8930 \\ 0.8839 \\ 0.8748 \\ 0.8659 \\ 0.8570 \\ 0.8483 \\ 0.8396 \\ 0.8310 \\ 0.8225 \\ 0.8141 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.969 \\ 0.959 \\ 0.957 \\ 0.967 \end{bmatrix} = \begin{bmatrix} 0.031 \\ 0.041 \\ 0.043 \\ 0.033 \end{bmatrix}$$

Multiply $(\mathbf{CWC}^{\mathbf{T}})^{-1}$ with $\mathbf{m}$-$\boldsymbol{\mu}$. The resulting vector represents the solution of the LSE that was set up in (14):

$$\boldsymbol{\zeta} = \begin{bmatrix} 58.6 \\ -34.1 \\ 11.8 \\ -5.7 \end{bmatrix}$$

To assess the discount function *P(t)* in arbitrary *t*, *t*>0, the Wilson functions $W(t,u_j)$, $j=1,2,3\ldots20$ have to be assessed and multiplied with **C**, as defined in (7). We want to compute the discount factor for *t*=4. We calculate $\mathbf{w}^{\mathbf{T}}=(W(4,u_j))_{j=1,2,3,..20}$, multiply it with $\mathbf{C}^{\mathbf{T}}$ and get the values of the kernel functions for *t*=4, i.e.:

$$(K_i(4))_{i=1,2,3,4} = \mathbf{w}^{\mathbf{T}}\mathbf{C}^{\mathbf{T}} = [0{,}027 \;\; 0{,}052 \;\; 0{,}075 \;\; 0{,}115]$$

From the linear combination of these kernel functions we get

$$(\mathbf{w}^{\mathbf{T}}\mathbf{C}^{\mathbf{T}})\boldsymbol{\zeta} = 0.0353$$

and adding the asymptotical factor $e^{-0.04114\times4} = 0.8483$, the discount function at maturity 4 years has the value *P(4)* = 0.0353 +0.8483=0.8836. This gives a spot rate (with annual compounding) of 3.141%.

9.4.3 Advantages and disadvantages of Smith-Wilson method

A trade-off has to be made between smoothness of the curve, accessibility, market consistency and transparency of the methodology. The Smith-Wilson method is seen as the macroeconomic technique that embodies best all of the above objectives. But also the Smith-Wilson method is far from perfect and has some weaknesses.

Below, the advantages and disadvantages of the Smith-Wilson approach that were taken into account when deciding on this method for Solvency II are summed up:

a) Both interpolation between the liquid market data points and extrapolation beyond the last data point have to be performed. For many methods (as for example for BarrieHibbert's method) interpolation and extrapolation are done separately, based on different principles and using different kinds of functions for assessing the different parts of the curve.

b) Smith-Wilson is based on solving a linear system of equations analytically. This is an advantage compared to all the methods that are based on e.g. minimizing sums of least square deviations, as these are susceptible to "catastrophic" jumps when the least-squares fit jumps from one set of parameters to another set of quite different values[30]. This problem is due to the non-linearity in the least squares formula which can give rise to more than one local minimum.

c) Smith-Wilson is sophisticated but still easy to use (in order to assess the term structure nothing else than an excel-sheet is needed), and gives both a relative smooth forward rate and a smooth spot rate curve in the extrapolated part.

d) The parameter *alpha*[31] has to be chosen outside the model. (The same is true for the parameter that determines the speed of convergence to the UFR in BarrieHibbert, and in Svensson or in Nelson-Siegel if they are used as macroeconomic models). Normally, expert judgment is needed to assess the convergence speed parameter for each currency separately. However, in order to have a harmonized approach over all currencies in Solvency II, the Level 2 text requires the estimated forward rates to be reasonably close to the UFR 40 years after last liquid market point. This condition has to be translated into appropriate speed parameters no matter which method is chosen.

   At BarrieHibbert the interpolation and extrapolation are done separately, based on different principles and using different kinds of functions for the different parts. They are able to define a parameter for the speed of convergence in the extrapolated part that is independent from the parameters they need in the interpolated part, whereas the alpha parameter in the Smith-Wilson approach determines both the speed of convergence to the

[30] For a thorough discussion of these problems see Andrew J. G. Cairns – "Descriptive Bond-Yield and Forward-rate models for the British Government Securities' Market" (1997). The paper can be found at http://www.ma.hw.ac.uk/~andrewc/papers/ajgc11.pdf

[31] Larger values of alpha give greater weight to the ultimate forward rate, while smaller values of alpha give more weight to the liquid market data.

UFR in the extrapolated part, and the smoothness of the curve in the interpolated part.

However, when the extrapolation is based on all liquid market data the smoothing in the interpolated part may not be as important as it is in the case where the interpolation is only based on a few knot points. On the other hand, due to the fact that only one method and one parameter are used for the whole term structure, we can expect more consistency both between the interpolated and extrapolated part of the same curve and over time for the curve as a whole[32].

e) There is no constraint forcing the discount function *P(t*) to decrease. In the liquid part of the assessed term structure we could have cases where *P(t)* is a decreasing function on the given liquid market data points, but becomes a locally increasing function on the interpolated curve. This can happen if a smooth curve is fitted between two neighbouring market data points for which the P(t) values are quite near. However, many other methods would have the same problem here. One way to deal with the problem is to take a smaller *alpha.* But as this could influence the extrapolation, the best approach to deal with the problem is to use expert judgment and delete one of the two market points from the data input.

f) Beyond the liquid part of the curve, *P(t)* may become negative. This is a disadvantage of S-W compared e.g. to parametric methods, as parametric methods often are based on formulas for the spot rate which per definition cannot produce negative discount functions. *P(t)* may become negative when the last forward rate in the liquid part of the curve is high compared to the sum of *UFR* and *alpha*. As an example, for *alpha* of 10% (i.e. *alpha*=0.1) and an UFR=4.2% the Smith-Wilson method doesn't work when interest rates are around 15% or higher. In this case, the *alpha* has to be increased. For *an alpha* of e.g. 20% (i.e. *alpha*=0.2) and an UFR=4.2%, the Smith-Wilson method doesn't work properly for interest rates of around and above 25%. If for certain sets of input market data *P(t)* will become negative, one has to take higher *alpha*s. This procedure can be made rules based.

[32] Imagine that - due to e.g. higher liquidity at the long end - the entry point for the extrapolation changes significantly from one period to the next period (valuation dates t1 and t2). In the BarrieHibbert method the rates for the same maturity as assessed at these two valuation dates t1 and t2 can become inconsistent, because for the same maturity two totally different methods and totally different parameters are applied, whereas this will not be the case in the S-W approach.

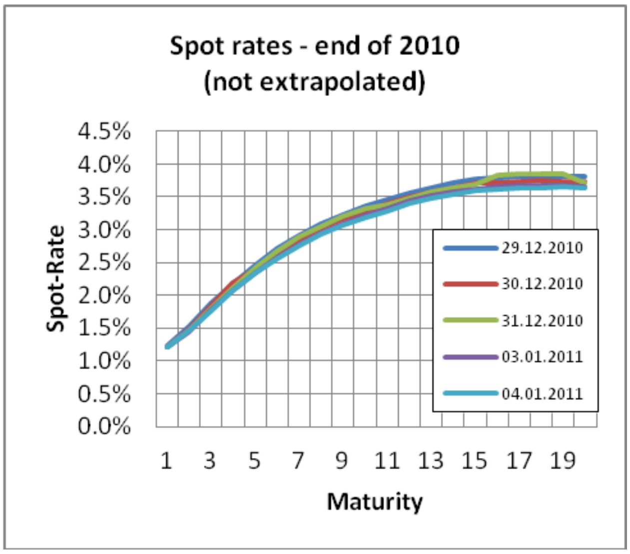
9.4.4 Analysis of the impact in the extrapolated part of the risk free curve with an example

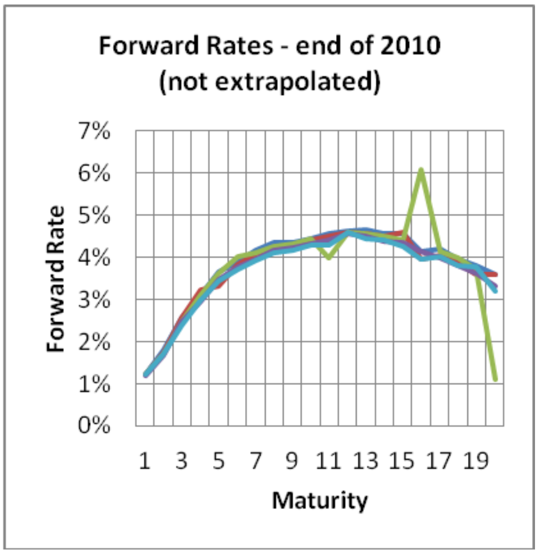
The following examples are intended to illustrate the impact of potential outliers on the risk-free rate curve.

The examples are based on the Euro and consider different points in time. The data is taken from Bloomberg. It is the spot rate for Interest Rates with different maturities, consistent with other considerations in the Issue Paper. To guarantee the convergence of the yield curves after 40 years, α=0.1 was chosen. The last liquid point was chosen as 20 years in the rest of analysis. As in QIS 5, the credit adjustment for the swap curve was assumed to be 10 BP.

In the first part of this section, we examine a number of yield curves and the connection between yield curves, forward rate curves and the extrapolated part of the curve. For this first example, we consider data at the end of 2010 – similar observations and conclusions could also be drawn for other time periods (see below).

For undertakings especially the spot rates are relevant. However the implied forward rates should also be considered, since these are the basis for the extrapolation mechanism applied by the Smith-Wilson technique. In the examples, the forward rates are calculated on basis of the non-arbitrage assumption from the spot rates and vice versa.

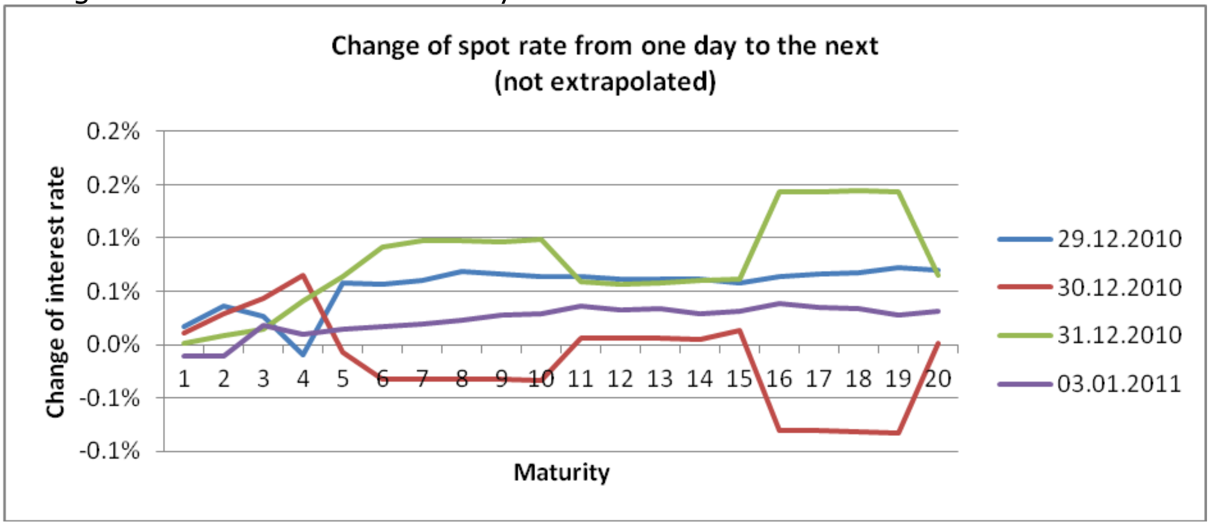




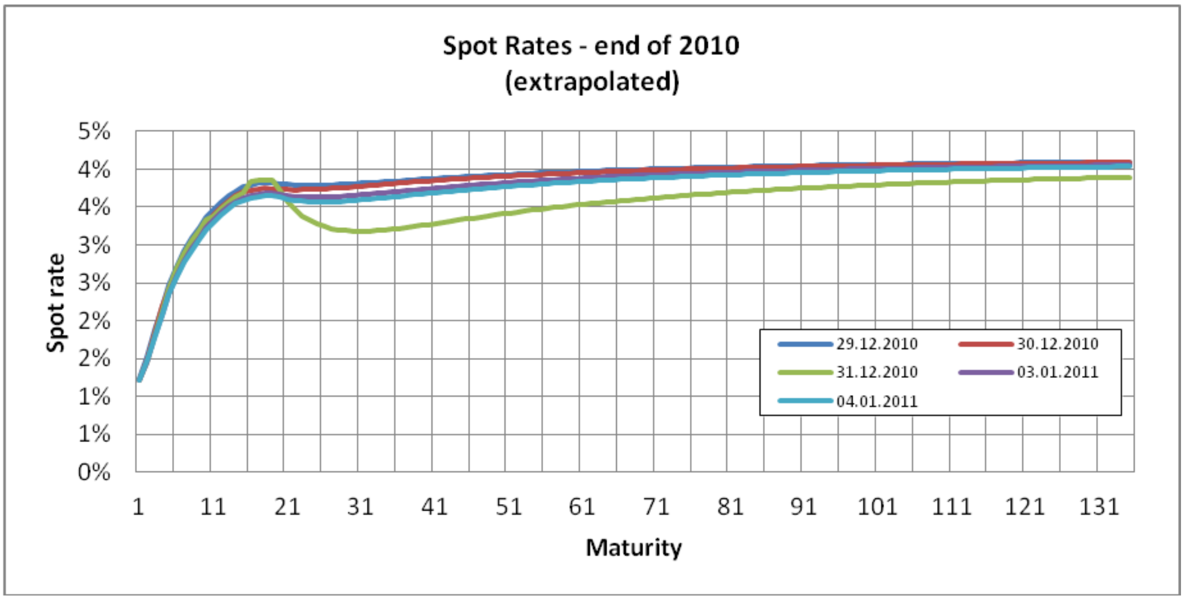
As illustrated by the diagrams above, when analysing the spot rates for 2010-12-31, some abnormalities can be observed. The one-year forward rate from year 19 to 20 measured 1% - this is a drastic change compared to the forward rate from year 18 to 19 which measured 3.5%. It also drastically differs from the one-year forward rate for the same year (19-20) just one day before or after year-end 2010.

Relying on such data would seem to contradict the expectation that the spot rates at high maturities should reflect long term expectations which shouldn't change on a daily basis and would thus have only a more limited impact on the volatility of the interest rates curve.

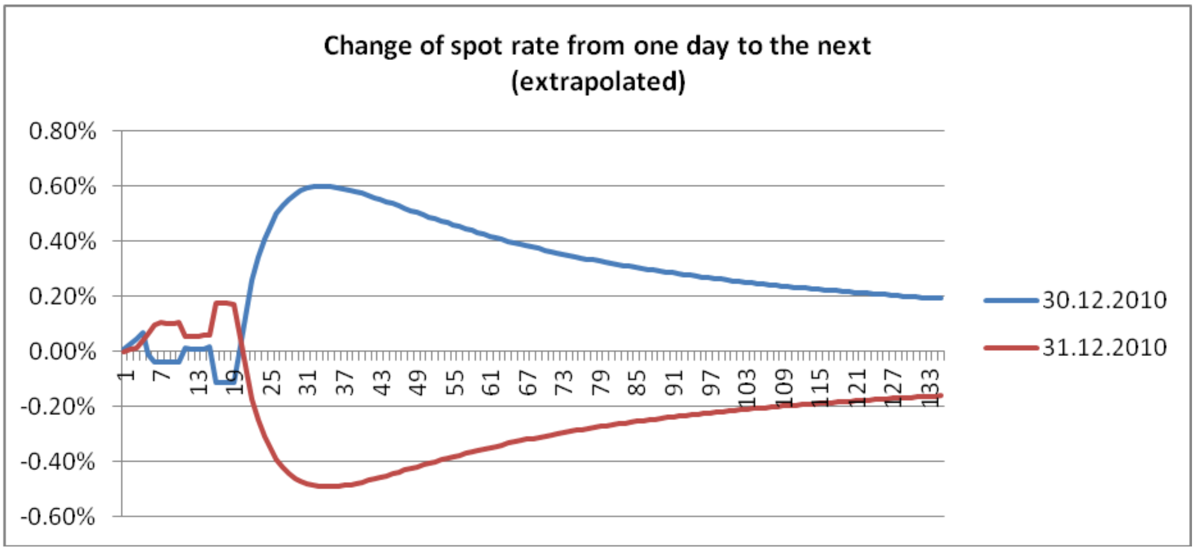
At the next (trading) day (i.e. at 3 January 2011) the interest rate curve has a less suspicious shape. The reason for this becomes clear when analyzing the changes between the different days:



As illustrated above, there is a strong change in the curve, which is then re-adjusted at the following day. Therefore, the forward rate from 19 to 20 years as at year-end 2010 should be seen as a temporary distortion. We'll show why this could be problematic by analysing the extrapolated part of the yield-curve:

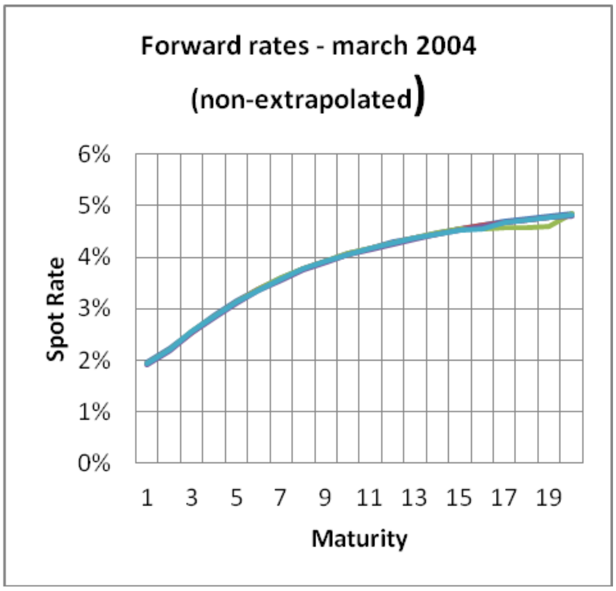


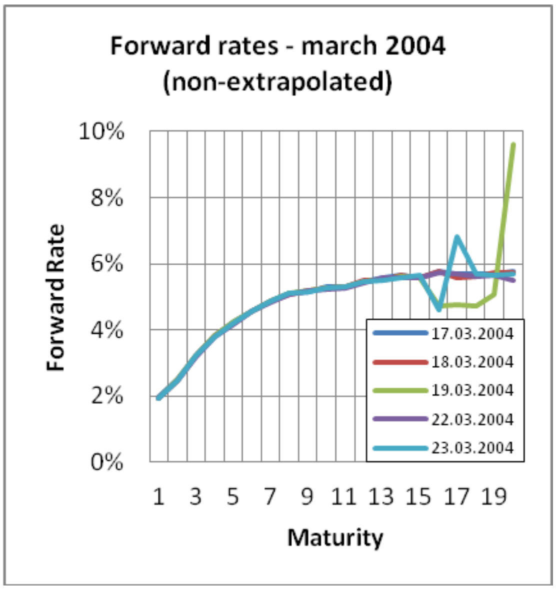
When analyzing the extrapolated part of the spot rates, it is clear that there is a big impact for exactly one day where the shape of the interest rate curve – especially in the extrapolated part - changes completely.

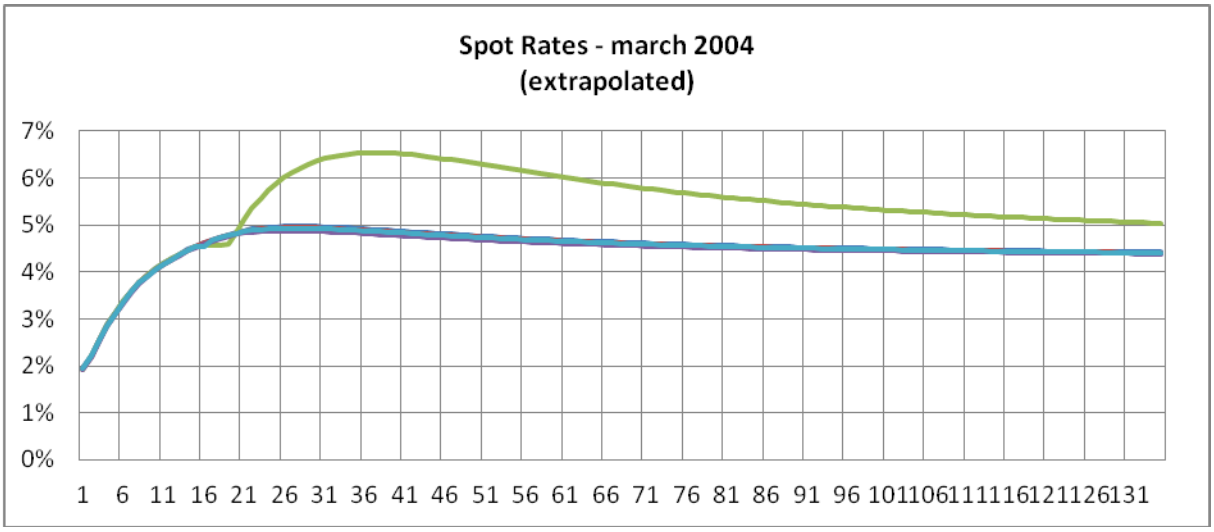


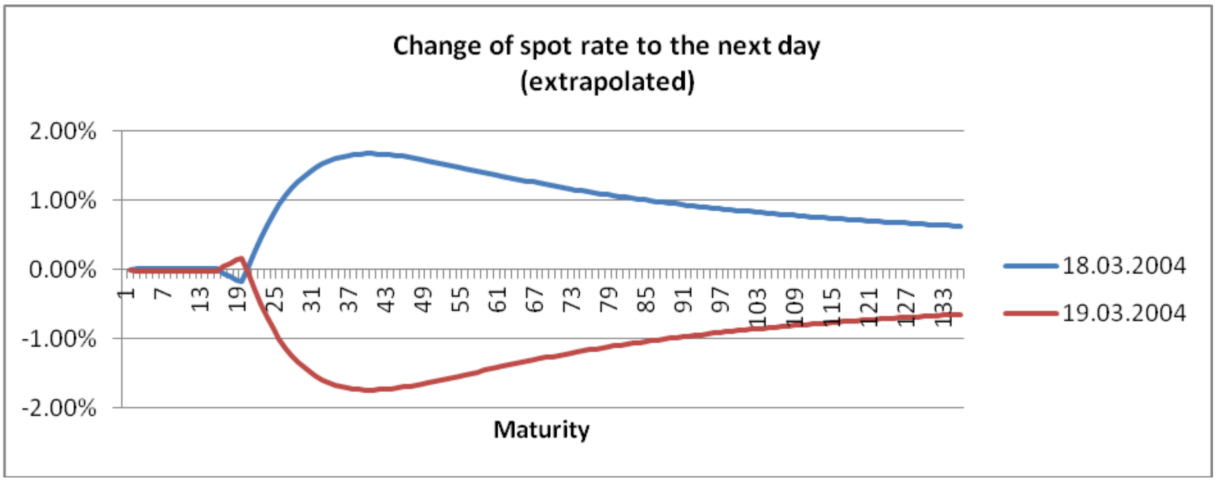
The change of the extrapolated part of the curve is bigger than the non-extrapolated part of the curve.

A further example is given below, which is based on data during March 2004:









Conclusions:

- In this example it is shown, that the extrapolated part strongly depends on the last 2 points and on the slope of the curve between the last 2 points.
- The volatility added in the extrapolated part could be larger than in the non-extrapolated part.

9.4.5 Estimation of expected real rate of interest

We expect that the real rates should not differ substantially across economies as far out as 100 years from now. Elroy Dimson, Paul Marsh and Mike Staunton provide a global comparison of annualized bond returns over the last 111 years (1900 to 2010) for the following 19 economies: Belgium, Italy, Germany, Finland, France, Spain, Ireland, Norway, Japan, Switzerland, Denmark, Netherlands, New Zealand, UK, Canada, US, South Africa, Sweden and Australia[33].

Figure 1: Real return on bonds 1900 – 2010

Source: Dimson, Marsh and Staunton – Credit Suisse Global Investment Returns Yearbook 2011

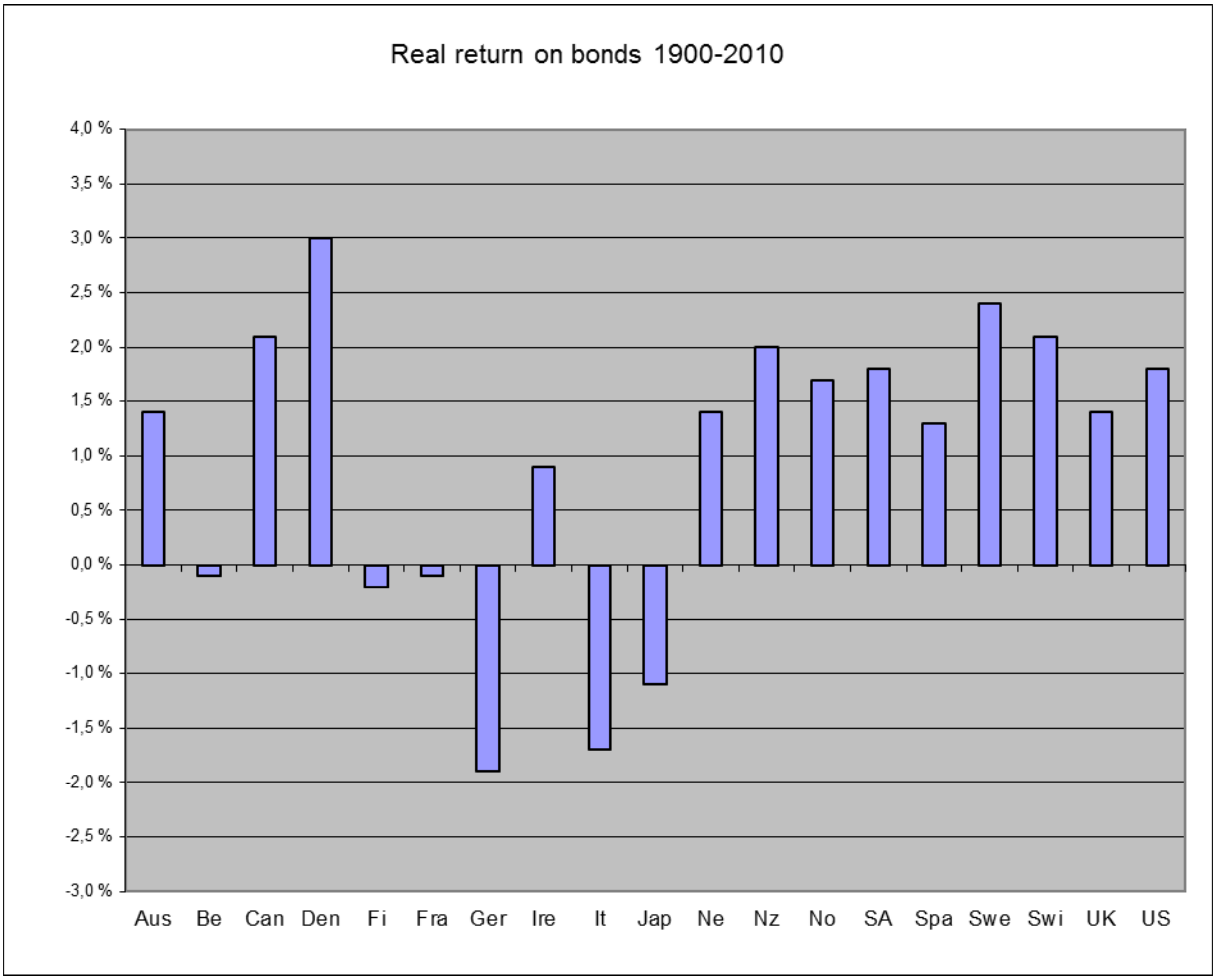


Figure 1 shows that, while in most countries bonds gave a positive real return, six countries experienced negative returns. Mostly the poor performance dates

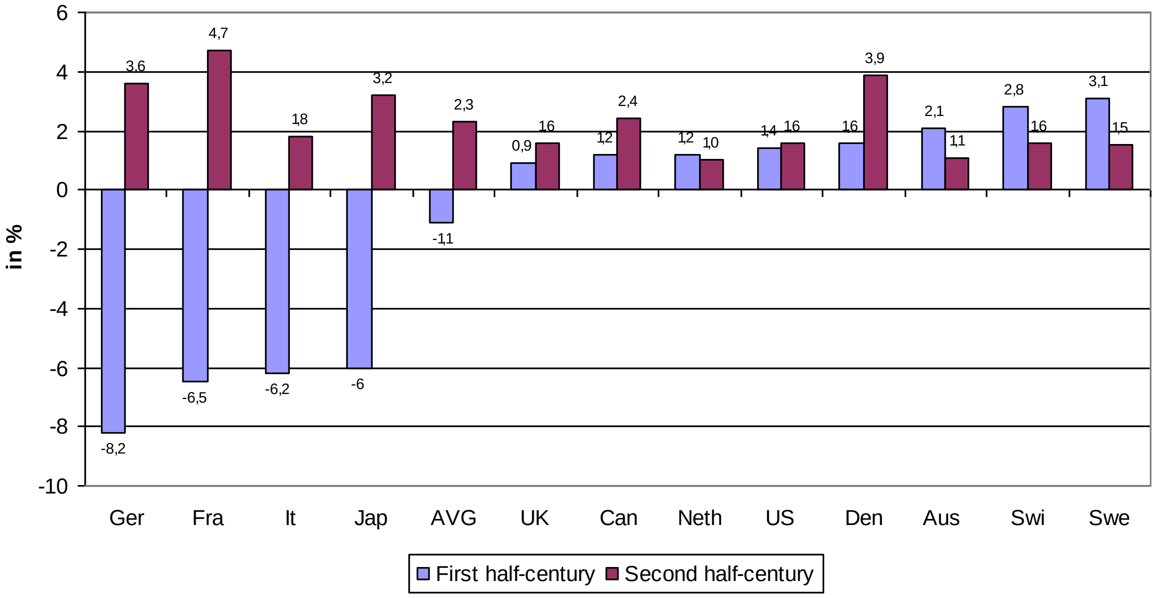
[33] Credit Suisse Global Investment Returns Yearbook 2011, To be found at https://infocus.credit-suisse.com/app/_customtags/download_tracker.cfm?dom=infocus.credit-suisse.com&doc=/data/_product_documents/_shop/300847/credit_suisse_global_investment_yearbook_2011.pdf&ts=20110326172226

back to the first half of the 20th century and can be explained with times of high or hyperinflation[34]. Aggregating the real returns on bonds for each currency[35] to an annual rate of real return on globally diversified bonds gives a rate of 1.6 per cent.

In an earlier publication, the same authors compared the real bond returns from the second versus the first half of the 20th century for the following 12 economies: Italy, Germany, France, Japan, Switzerland, Denmark, Netherlands, UK, Canada, US, Sweden and Australia[36]. The average real bond return over the second half of the 20th century was computed as annually 2.3 per cent (compared to -1.1 percent for the first half of the 20th century).

*Figure 2: Real bond returns: first versus second half of 20th century**

Source: Dimson, Marsh and Staunton (ABN-Amro/London Business School)



* Data for Germany excludes 1922-23. AVG = Average

In light of the above data, 2.2 per cent, the expected real interest rate that was assessed for QIS5 continues to be an adequate estimate for the expected real interest rate.

[34] German hyperinflation in 1922/1923, in Italy an inflation of 344% in 1944, in France 74% in 1946 and in Japan 317% in 1946.

[35] Average where each return is weighted by its country's GDP.

[36] Elroy Dimson, Paul Marsh and Mike Staunton: Risk and return in the 20th and 21th, Business Strategy Review, 2000, Volume 11 issue 2, pp 1-18. See Figure 4 on page 5. The article can be downloaded at: http://docs.google.com/viewer?a=v&q=cache:07V7vM0gu5oJ:citeseerx.ist.psu.edu/viewdoc/download%3Fdoi%3D10.1.1.11.7613%26rep%3Drep1%26type%3Dpdf+Risk+and+return+in+the+20th+and+21th+Centuries&hl=no&gl=no&sig=AHIEtbQbxwuXZNO6ViVIqkV0KZ63LKhB0g

9.5 Government bonds and swap rates for 35 currencies

			Maximum bond term available			Maximum swap term available					
			EIOPA		CRO Forum	EIOPA	CRO Forum	Bloomberg ticker		Availability of data	
No.	**Currency**	**Publication bucket[1]**	**Government bond**	**Generic bond**	**Government bond**	**Term to maturity**	**Term to maturity**	**Generic bond ticker**	**Swap ticker**	**Data for at least 5 years is available**	**Notes**
1	Algerian Dinar DZD		-	-	-	-	-	-	-		No data could be sourced from Bloomberg.
2	Argentine Peso ARS		34.1	-	-	-	-	-	-		The government bond referred here is a sinkable bond and pays fixed coupon denominated in ARS.
3	Australian Dollar AUD		18.8	15	-	30	-	GACGBTT	AD-SWAPTT	√	Data on swaps is available for a period over 10 years where as for generic government bonds data is available for a period over 20 years.
4	Brazilian Real BRL		29.2	30	-	5	-	GEBUTT	BCSW*PD	√	Data on swaps is available for a period over 10 years. The data on generic bonds is available for a sufficiently long period but for bonds of term to maturity of 9 years or less we were able to source data only from 2008.
5	British Pound GBP		48.2	30	49.8	50	50	GUKGTT	BPSWTT	√	The data on swaps is available for a period over 10 years except for the swaps of term 45 and 50 years where the data is available from year 2003. For generic government bonds data is available for a period over 10 years.
6	Bulgarian Lev BGN		13.8	-	9.1	10	10	-	BLSATT	√	Data on swaps is available from Oct 2005
7	Canadian Dollar CAD		34.1	30	-	50	-	GCANTT	CDSWTT		We were unable to source data on 40 and 50 year swaps from Bloomberg but tickers were available. However, data on swaps of shorter term to maturity than 40 year swap and generic government bonds is available for a sufficiently large period.
8	Chilean Peso CLP		10.8	-	-	10	-	-	CHSWPTT	√	
9	Chinese Yuan CNY		49.5	10	-	10	-	GCNYTT	CCSWOTT		Data on swaps is available from May 2006 and appears to be updated daily. Data on generic government bonds is available from June 2005.
10	Colombian Peso COP		14.8	15	-	30	-	COGRTT	CLSWUTT		Data on 30 year swap is available from 2008 but it does not appear to be updated frequently. For the rest it is available from 2005/2007. Data on generic bonds is available for a period over 10 years and appears to be more frequently updated.
11	Croatian Kuna HRK		10.7	7	-	-	-	HRGTT	-		There are are a number of discontinuities in the data.
12	Czech Koruna CZK		46	30	47.8	30	30	CZGBTT	CKSWTT	√	Data on swaps is available for a period of over 5 years. Data on 30 year generic government bond is available from 2006 where as for the rest it is available for a longer period. However, at certain intervals the data does not seem to be available.
13	Denmark Krone DKK		28	30	-	30	-	GDGBTT	DKSWTT	√	Data on swaps for all maturities is available for a period over 10 years. The data on generic government bond of 30 years is available from 1994 but data for the period 1st Jan 2005-1st Dec 2008 for this bond is missing. Similarly, there is discontinuity in the rest of the data.
14	Euro EUR		48.4	30	45.3	50	50	GECUTT	EUSATT	√	Data on both swaps and government bonds is available for a sufficiently long period. The government bond of term to maturity of 48.4 years if French government bond.
15	Hungarian Forint HUF		16.9	15	13.9	20	20	GHGBTT	HFSWTT	√	Data on both swaps and generic bonds is available for all terms to maturity for a period over 10 years
16	Icelandic Krona ISK		19.2	6	15.3	7	5	GRICT	IKSWTT	√	Data on swaps is available from year 2005-2006. For generic government bonds data is available year 2007-2008 but it does not appear to be updated frequently.
17	Japanese Yen JPY		39.3	40	26.7	50	50	GJGBTT	JYSWTT	√	Data on swaps for terms 40 and 50 year is available from 2007 but for other terms to maturity it is available for a sufficiently large period. The data on generic government bonds is available for a period of over 10 years except for the 40 year bond where it is available from 2008.
18	Latvian Lats LVL		9.3	-	-	-	-	-	-		Limited data on government bonds is available.
19	Lithuanian Litas LTL		6.3	-	-	-	-	-	-		Data on bond with 6.8 years term to maturity is only available at 29th April 2011. More data is available on other bonds but does not appear to be updated frequently.

20	Mexican Peso MXN	27	30	-	20	-	GMXNTT	MPSWT T	√	Data on swaps and generic bonds is available for a sufficiently large period. The format of Bloomberg ticker for swaps of term to maturity less than 15 years is slightly different as there is an additional alphabet at the end.
21	Moroccan Dirham MAD	25.1	-	-	10	-	-	MDSWT T		No data before 2007 is available. The data on 10 year swap is only available from 2008 and for other maturities it has not been updated since May 2010.
22	New Zea-land Dol-lar NZD	11.4	10	-	25	-	GNZGBTT	NDSWT T	√	Data on swaps is available for at least 10 years where as data on generic bonds are available for a period over 25 years. We are able to find ticker for the 25 year swap but we were unable to source any data for it.
23	Norwegian Krone NOK	9.5	10	9.4	30	30	GNORTT	NKSWT T	√	Data on swaps for all maturities is available for a period over 10 years. For the generic govern-ment bonds data is available for a period over 10 years but there is discontinuity in data.
24	Polish Zloty PLN	25.4	20	27.3	30	20	POGBTT	PZSWT T	√	Data on swaps of 30 years term to maturity is available from April 2007 where as for other maturities it is available for a sufficiently long period. Data on generic government bonds is available for a sufficiently long period except for the 15 year bond where it is available from July 2007.
25	Romanian Leu RON	9.6	5	10.5	20	20	GRROT	RNS-WTT		Data on 12-20 year swaps is available from February 2007 where as for the rest it is available from October 2006.
26	Russian Rubles RUB	38	25	-	10	-	MICXRUT T	RRSWM TT		Data on swaps is available from April/Aug 2006 and appears to be frequently updated. Data on generic bonds is only available from year 2009/2010.
27	Singapo-rean Dol-lar SGD	18.8	20	-	30	-	MASBTT	SDSWT T	√	Data on swaps is available for a sufficiently large period except for the 20 year and 30 year swaps where it is available from 2006 and 2008 respectively. Data on generic bonds with 20 year term to maturity is available from 2007 but for the rest it is available for a sufficiently large period.
28	South African Rand ZAR	29.3	30	-	30	-	GSABTT	SASWT T	√	
29	South Korean Won KRW	19.1	20	-	20	-	GVSKTT	KWSW OTT	√	Data on swaps and generic bonds is available for a sufficiently large period.
30	Swedish Krona SEK	27.3	30	29.2	30	30	GSGB30	SKSWT T	√	Data on swaps for all maturities is available for a period over 10 years. The data on generic government bonds for 30 year maturity is available from March 2009, where as for other ma-turities it is available for a sufficiently long period.
31	Swiss Franc CHF	37.2	30	39	50	50	GSWIS-STT	SFSWT T	√	The data on 50 year swap is available from April 2005, where as data on the rest of the swaps including 40 year swaps is available for a period over 10 years. The data on generic govern-ment bonds is available for a period of over 10 years.
32	Tunisian Dinar TND	10.8	-	-	-	-	-	-		We were unable to source any data on swaps.
33	Turkish Lira TRY	9.7	10	4.4	10	10	IECMTT	TYSWT T	√	Data on swaps is available from May 2005.
34	US Dollar USD	30	30	29.9	50	50	USGGTT	USS-WTT	√	Data for the 50 year swap is available from 2004 but for the 45 year term to maturity the data is available from 2009. For the generic government bonds data is available from 1977. In addi-tion, GS index for generic government bonds is also available.
35	Venezue-lan Bolivar VEB	8.9	-	-	-	-	-	-		Only data on government bonds is available.

*may contain letter A, B, C, D, E, F, I, K, L, M or N

[1] Two buckets of currencies will be defined that will determine the frequency of EIOPA publication

9.6 Basic risk free rate curves for EUR, GBP and USD

The zero-coupon curves contained in this annex have been calculated following the specifications of the current issues paper, hence observed swap rates are used as a calculation basis; forward rates are adjusted for credit risk using a deduction of 10bp for all maturities; rates for unobserved maturity points are interpolated using the Smith-Wilson model and the Smith-Wilson model is also used for extrapolation.

A more detailed description of the applied process is summarised by the following steps:

1) Swap rates are used to calculate zero coupon rates. This is done in Matlab using the "zbtprice" function;
2) The zero rates are used to calculate forward rates. This is done in Matlab using the "zero2fwd" function;
3) The forward rates are adjusted for credit risk by subtracting 10bp from each maturity;
4) Credit adjusted zero curves are calculated from the forward rates. This is done in Matlab using the "fwd2zero" function;
5) The Smith-Wilson model is estimated on the credit adjusted zero rates and interpolation for possibly unobserved maturity points and extrapolated below the 1Y maturity and beyond the last observed maturity, is achieved using the Smith-Wilson technique. This is done in Matlab via a user defined function;
6) The speed of convergence (alpha parameter) in the Smith-Wilson model is set to the value of 0.10. If convergence to the UFR of the instantaneous forward rate is not achieved within 40 years after the last liquid point, then the alpha parameter is reoptimised to ensure convergence within the required time-frame;
7) There are no other adjustments made to the curve;

In addition, the following parameter choices are made:

a) number of coupon payments per year is set to 1;
b) compounding is set to annual (i.e. annual discrete compounding is used);
c) the day count convention is set to 30/360 European;
d) Ultimate forward rate is set to 4.2% for the curves included in this excel workbook;
e) convergence to the ultimate forward rate (UFR), within 3bp, should happen within 40 years after the last liquid point;
f) calculated forward rates are adjusted downwards for all maturities by 10bp to account for credit risk;
g) The last liquid points and the maturities used in the calculations for each included currency can be seen in the "Curves" sheet contained in the current excel workbook.

OBSERVED SWAP DATA

EUR SWAP used maturities

(years) Maturity	swap 12/30/2011	(years) Maturity	swap 12/31/2010
1	1.4070	1	1.2960
2	1.3050	2	1.5220
3	1.3560	3	1.8560
4	1.5210	4	2.1690
5	1.7090	5	2.4760
6	1.8930	6	2.6690
7	2.0500	7	2.9280
8	2.1730	8	3.0850
9	2.2760	9	3.2130
10	2.3660	10	3.3240
12	2.5245	12	3.4630
15	2.6470	15	3.6250
20	2.6710	20	3.6840

UK SWAP used maturities

(years) Maturity	swap 12/30/2011	(years) Maturity	swap 12/31/2010
1	1.3432	1	1.1175
2	1.3070	2	1.4875
3	1.3420	3	1.9410
4	1.4210	4	2.2975
5	1.5440	5	2.6050
6	1.6990	6	2.8760
7	1.8560	7	3.0920
8	2.0170	8	3.2670
9	2.1490	9	3.4040
10	2.2700	10	3.5260
11	2.3730	11	3.6400
12	2.4580	12	3.6990
13	2.5350	13	3.7900
14	2.5950	14	3.8450
15	2.6440	15	3.8590
16	2.6850	16	3.9150
17	2.7250	17	3.9300
18	2.7603	18	3.9450
19	2.7900	19	3.9500
20	2.8210	20	3.9300
25	2.9330	25	3.9330
30	2.9880	30	3.9000
40	3.0380	40	3.8300
50	3.0780	50	3.8030

US SWAP used maturities

(years) Maturity	swap 12/30/2011	(years) Maturity	swap 12/31/2010
1	0.6760	1	0.4280
2	0.7060	2	0.7840
3	0.8132	3	1.2743
4	0.9996	4	1.7270
5	1.2220	5	2.1540
6	1.4423	6	2.5160
7	1.6361	7	2.8070
8	1.7959	8	3.0220
9	1.9256	9	3.2040
10	2.0200	10	3.3570
11	2.1160	11	3.5100
12	2.2154	12	3.5920
13	2.2730	13	3.7000
14	2.3360	14	3.7700
15	2.3899	15	3.8438
16	2.4160	16	3.8900
17	2.4458	17	3.9280
18	2.5010	18	3.9590
19	2.5200	19	3.9840
20	2.5093	20	3.9800
25	2.5663	25	4.0500
30	2.6020	30	4.0858

Bid observations are used.
Bloomberg codes used (the same as in the QIS5 documentation):

- Euro: EUSAYY Curncy
- GB: BPSWYY Curncy
- US: USSWYY Curncy

CALCULATED SMITH-WILSON ZERO COUPON CURVES - SMITH WILSON INTERPOLATION

EUR 12/30/2011 Maturity	Curve	Forward	EUR 12/31/2010 Maturity	Curve	Forward	UK 12/30/2011 Maturity	Curve	Forward
0.08	1.3539	1.3539	0.08	1.0983	1.0983	0.08	1.2498	1.2498
0.17	1.3554	1.3569	0.17	1.1034	1.1083	0.17	1.2523	1.2549
0.25	1.3557	1.3562	0.25	1.1092	1.1211	0.25	1.2542	1.2576
0.33	1.3548	1.3521	0.33	1.1158	1.1358	0.33	1.2555	1.2593
0.42	1.3528	1.3450	0.42	1.1232	1.1529	0.42	1.2561	1.2584
0.50	1.3496	1.3332	0.50	1.1313	1.1723	0.50	1.2561	1.2560
0.58	1.3453	1.3196	0.58	1.1402	1.1958	0.58	1.2554	1.2517
0.67	1.3399	1.3019	0.67	1.1499	1.2175	0.67	1.2542	1.2455
0.75	1.3333	1.2792	0.75	1.1603	1.2436	0.75	1.2523	1.2370
0.83	1.3256	1.2592	0.83	1.1715	1.2719	0.83	1.2499	1.2285
0.92	1.3168	1.2261	0.92	1.1834	1.3024	0.92	1.2468	1.2151

UK 12/31/2010 Maturity	Curve	Forward	US 12/30/2011 Maturity	Curve	Forward	US 12/31/2010 Maturity	Curve	Forward
0.08	0.8528	0.8528	0.08	0.5418	0.5418	0.08	0.1643	0.1643
0.17	0.8597	0.8665	0.17	0.5467	0.5515	0.17	0.1729	0.1811
0.25	0.8684	0.8859	0.25	0.5511	0.5593	0.25	0.1827	0.2027
0.33	0.8786	0.9096	0.33	0.5553	0.5681	0.33	0.1938	0.2275
0.42	0.8904	0.9382	0.42	0.5590	0.5735	0.42	0.2062	0.2562
0.50	0.9039	0.9715	0.50	0.5625	0.5802	0.50	0.2199	0.2887
0.58	0.9189	1.0127	0.58	0.5656	0.5840	0.58	0.2348	0.3281
0.67	0.9355	1.0518	0.67	0.5683	0.5875	0.67	0.2510	0.3643
0.75	0.9537	1.0992	0.75	0.5707	0.5906	0.75	0.2684	0.4079
0.83	0.9734	1.1512	0.83	0.5728	0.5908	0.83	0.2871	0.4550
0.92	0.9947	1.2077	0.92	0.5745	0.5926	0.92	0.3069	0.5058

CALCULATED SMITH-WILSON ZERO COUPON CURVES - SMITH WILSON INTERPOLATION

EUR
12/30/2011

Maturity	Curve	Forward
1	1.3069	1.2022
2	1.2043	1.1021
3	1.2562	1.3600
4	1.4250	1.9325
5	1.6191	2.4012
6	1.8114	2.7773
7	1.9773	2.9775
8	2.1084	3.0300
9	2.2192	3.1118
10	2.3171	3.2012
11	2.4107	3.3509
12	2.4929	3.4005
13	2.5565	3.3252
14	2.6012	3.1832
15	2.6276	2.9980
16	2.6378	2.7909
17	2.6385	2.6498
18	2.6359	2.5915
19	2.6346	2.6105
20	2.6380	2.7022
21	2.6480	2.8494
22	2.6636	2.9914
23	2.6833	3.1177
24	2.7060	3.2297
25	2.7309	3.3310
26	2.7573	3.4179
27	2.7846	3.4971
28	2.8125	3.5678
29	2.8406	3.6332
30	2.8688	3.6875
31	2.8967	3.7383
32	2.9243	3.7838
33	2.9515	3.8272
34	2.9782	3.8614
35	3.0043	3.8945
36	3.0297	3.9242
37	3.0546	3.9535
38	3.0787	3.9751
39	3.1022	3.9969
40	3.1249	4.0165
41	3.1470	4.0366
42	3.1684	4.0500
43	3.1892	4.0644
44	3.2093	4.0775
45	3.2288	4.0916
46	3.2477	4.0997
47	3.2660	4.1093
48	3.2836	4.1180
49	3.3008	4.1281
50	3.3174	4.1328
51	3.3334	4.1392
52	3.3490	4.1450
53	3.3641	4.1524
54	3.3787	4.1549
55	3.3928	4.1592
56	3.4065	4.1630
57	3.4198	4.1686
58	3.4327	4.1696
59	3.4452	4.1725
60	3.4573	4.1751
61	3.4691	4.1794
62	3.4805	4.1795
63	3.4916	4.1814
64	3.5024	4.1832
65	3.5129	4.1866
66	3.5231	4.1861
67	3.5330	4.1874
68	3.5426	4.1886
69	3.5519	4.1914
70	3.5610	4.1906
71	3.5699	4.1914
72	3.5785	4.1922
73	3.5869	4.1946
74	3.5951	4.1936
75	3.6031	4.1941

EUR
12/31/2010

Maturity	Curve	Forward
1	1.1960	1.3351
2	1.4259	1.6562
3	1.7654	2.4480
4	2.0877	3.0608
5	2.4091	3.7045
6	2.6125	3.6383
7	2.8937	4.5968
8	3.0645	4.2678
9	3.2054	4.3390
10	3.3294	4.4548
11	3.4148	4.2723
12	3.4835	4.2423
13	3.5546	4.4101
14	3.6204	4.4826
15	3.6707	4.3771
16	3.7000	4.1404
17	3.7137	3.9320
18	3.7180	3.7911
19	3.7178	3.7152
20	3.7171	3.7031
21	3.7183	3.7422
22	3.7214	3.7876
23	3.7260	3.8279
24	3.7318	3.8644
25	3.7384	3.8972
26	3.7457	3.9272
27	3.7534	3.9533
28	3.7613	3.9772
29	3.7695	3.9988
30	3.7778	4.0189
31	3.7861	4.0358
32	3.7944	4.0516
33	3.8026	4.0659
34	3.8108	4.0795
35	3.8187	4.0904
36	3.8266	4.1009
37	3.8342	4.1104
38	3.8417	4.1197
39	3.8490	4.1267
40	3.8561	4.1337
41	3.8631	4.1400
42	3.8698	4.1465
43	3.8763	4.1509
44	3.8827	4.1556
45	3.8888	4.1598
46	3.8948	4.1644
47	3.9006	4.1671
48	3.9062	4.1702
49	3.9116	4.1730
50	3.9169	4.1763
51	3.9220	4.1779
52	3.9270	4.1800
53	3.9318	4.1819
54	3.9364	4.1843
55	3.9410	4.1852
56	3.9453	4.1866
57	3.9496	4.1878
58	3.9537	4.1896
59	3.9577	4.1900
60	3.9616	4.1909
61	3.9654	4.1918
62	3.9690	4.1932
63	3.9726	4.1933
64	3.9761	4.1939
65	3.9794	4.1945
66	3.9827	4.1956
67	3.9859	4.1954
68	3.9889	4.1959
69	3.9920	4.1962
70	3.9949	4.1972
71	3.9977	4.1969
72	4.0005	4.1972
73	4.0032	4.1974
74	4.0058	4.1982
75	4.0084	4.1979

UK
12/30/2011

Maturity	Curve	Forward
1	1.2431	1.2042
2	1.2067	1.1704
3	1.2423	1.3133
4	1.3229	1.5650
5	1.4495	1.9586
6	1.6109	2.4205
7	1.7763	2.7735
8	1.9484	3.1607
9	2.0911	3.2423
10	2.2238	3.4243
11	2.3379	3.4857
12	2.4329	3.4826
13	2.5201	3.5739
14	2.5880	3.4737
15	2.6434	3.4222
16	2.6898	3.3879
17	2.7360	3.4798
18	2.7770	3.4762
19	2.8114	3.4314
20	2.8485	3.5553
21	2.8832	3.5822
22	2.9140	3.5618
23	2.9410	3.5373
24	2.9646	3.5065
25	2.9847	3.4707
26	3.0017	3.4277
27	3.0161	3.3895
28	3.0282	3.3563
29	3.0385	3.3284
30	3.0473	3.3029
31	3.0549	3.2820
32	3.0615	3.2652
33	3.0673	3.2527
34	3.0724	3.2427
35	3.0771	3.2364
36	3.0814	3.2331
37	3.0855	3.2329
38	3.0894	3.2344
39	3.0933	3.2387
40	3.0971	3.2453
41	3.1009	3.2549
42	3.1049	3.2684
43	3.1091	3.2874
44	3.1137	3.3116
45	3.1188	3.3414
46	3.1243	3.3750
47	3.1305	3.4146
48	3.1374	3.4597
49	3.1450	3.5115
50	3.1534	3.5673
51	3.1627	3.6285
52	3.1727	3.6854
53	3.1834	3.7379
54	3.1944	3.7821
55	3.2058	3.8232
56	3.2175	3.8601
57	3.2293	3.8951
58	3.2412	3.9231
59	3.2532	3.9500
60	3.2652	3.9743
61	3.2772	3.9981
62	3.2890	4.0157
63	3.3008	4.0335
64	3.3125	4.0495
65	3.3240	4.0660
66	3.3354	4.0770
67	3.3466	4.0888
68	3.3576	4.0994
69	3.3685	4.1111
70	3.3792	4.1177
71	3.3897	4.1255
72	3.3999	4.1326
73	3.4100	4.1410
74	3.4199	4.1448
75	3.4296	4.1500

UK
12/31/2010

Maturity	Curve	Forward
1	1.0175	1.2688
2	1.3923	1.7686
3	1.8542	2.7841
4	2.2219	3.3331
5	2.5440	3.8427
6	2.8329	4.2935
7	3.0665	4.4787
8	3.2584	4.6115
9	3.4102	4.6318
10	3.5478	4.7978
11	3.6793	5.0031
12	3.7436	4.4523
13	3.8527	5.1714
14	3.9160	4.7439
15	3.9241	4.0376
16	3.9939	5.0457
17	4.0054	4.1894
18	4.0182	4.2365
19	4.0167	3.9907
20	3.9771	3.2277
21	3.9512	3.4350
22	3.9442	3.7955
23	3.9467	4.0033
24	3.9515	4.0613
25	3.9523	3.9717
26	3.9452	3.7672
27	3.9316	3.5794
28	3.9139	3.4374
29	3.8941	3.3396
30	3.8736	3.2820
31	3.8538	3.2620
32	3.8349	3.2501
33	3.8169	3.2417
34	3.7997	3.2350
35	3.7835	3.2345
36	3.7683	3.2351
37	3.7539	3.2384
38	3.7404	3.2427
39	3.7279	3.2522
40	3.7162	3.2624
41	3.7054	3.2753
42	3.6956	3.2910
43	3.6867	3.3134
44	3.6787	3.3390
45	3.6718	3.3691
46	3.6660	3.4029
47	3.6612	3.4429
48	3.6576	3.4870
49	3.6551	3.5364
50	3.6539	3.5912
51	3.6538	3.6502
52	3.6548	3.7048
53	3.6566	3.7538
54	3.6593	3.7982
55	3.6625	3.8373
56	3.6662	3.8728
57	3.6704	3.9048
58	3.6750	3.9342
59	3.6798	3.9594
60	3.6848	3.9827
61	3.6900	4.0037
62	3.6954	4.0236
63	3.7009	4.0398
64	3.7064	4.0552
65	3.7120	4.0691
66	3.7176	4.0827
67	3.7232	4.0930
68	3.7287	4.1033
69	3.7343	4.1125
70	3.7398	4.1219
71	3.7453	4.1284
72	3.7507	4.1353
73	3.7560	4.1414
74	3.7613	4.1481
75	3.7665	4.1520

US
12/30/2011

Maturity	Curve	Forward
1	0.5760	0.5910
2	0.6061	0.6361
3	0.7141	0.9303
4	0.9033	1.4724
5	1.1312	2.0497
6	1.3596	2.5081
7	1.5629	2.7907
8	1.7325	2.9263
9	1.8714	2.9919
10	1.9728	2.8888
11	2.0777	3.1322
12	2.1885	3.4146
13	2.2514	3.0108
14	2.3222	3.2455
15	2.3830	3.2381
16	2.4100	2.8154
17	2.4426	2.9657
18	2.5091	3.6463
19	2.5287	2.8810
20	2.5088	2.1330
21	2.5023	2.3706
22	2.5121	2.7200
23	2.5303	2.9304
24	2.5503	3.0117
25	2.5671	2.9716
26	2.5777	2.8425
27	2.5841	2.7516
28	2.5892	2.7243
29	2.5949	2.7574
30	2.6033	2.8462
31	2.6154	2.9785
32	2.6307	3.1062
33	2.6485	3.2211
34	2.6682	3.3202
35	2.6894	3.4098
36	2.7115	3.4898
37	2.7344	3.5636
38	2.7578	3.6251
39	2.7814	3.6823
40	2.8051	3.7336
41	2.8288	3.7823
42	2.8523	3.8209
43	2.8756	3.8580
44	2.8986	3.8914
45	2.9212	3.9243
46	2.9435	3.9485
47	2.9653	3.9728
48	2.9866	3.9948
49	3.0075	4.0174
50	3.0279	4.0324
51	3.0479	4.0485
52	3.0673	4.0631
53	3.0863	4.0790
54	3.1047	4.0880
55	3.1227	4.0987
56	3.1403	4.1084
57	3.1573	4.1198
58	3.1740	4.1250
59	3.1901	4.1321
60	3.2059	4.1385
61	3.2212	4.1470
62	3.2361	4.1496
63	3.2507	4.1544
64	3.2648	4.1587
65	3.2786	4.1651
66	3.2920	4.1661
67	3.3050	4.1693
68	3.3177	4.1722
69	3.3301	4.1771
70	3.3422	4.1771
71	3.3539	4.1792
72	3.3654	4.1812
73	3.3765	4.1852
74	3.3874	4.1845
75	3.3980	4.1859

US
12/31/2010

Maturity	Curve	Forward
1	0.3280	0.5601
2	0.6865	1.0463
3	1.1833	2.1844
4	1.6483	3.0560
5	2.0947	3.8999
6	2.4803	4.4356
7	2.7957	4.7079
8	3.0315	4.6965
9	3.2344	4.8718
10	3.4077	4.9840
11	3.5862	5.3876
12	3.6777	4.6886
13	3.8074	5.3762
14	3.8889	4.9563
15	3.9788	5.2460
16	4.0316	4.8252
17	4.0743	4.7607
18	4.1084	4.6910
19	4.1348	4.6107
20	4.1158	3.7550
21	4.1076	3.9442
22	4.1186	4.3502
23	4.1399	4.6104
24	4.1644	4.7292
25	4.1860	4.7044
26	4.2005	4.5648
27	4.2089	4.4277
28	4.2131	4.3260
29	4.2146	4.2580
30	4.2149	4.2233
31	4.2150	4.2161
32	4.2150	4.2146
33	4.2149	4.2132
34	4.2148	4.2120
35	4.2147	4.2108
36	4.2146	4.2098
37	4.2144	4.2089
38	4.2143	4.2080
39	4.2141	4.2073
40	4.2139	4.2066
41	4.2137	4.2059
42	4.2135	4.2054
43	4.2133	4.2049
44	4.2131	4.2044
45	4.2129	4.2040
46	4.2127	4.2036
47	4.2125	4.2033
48	4.2123	4.2030
49	4.2121	4.2027
50	4.2119	4.2024
51	4.2117	4.2022
52	4.2115	4.2020
53	4.2113	4.2018
54	4.2112	4.2016
55	4.2110	4.2015
56	4.2108	4.2013
57	4.2106	4.2012
58	4.2105	4.2011
59	4.2103	4.2010
60	4.2102	4.2009
61	4.2100	4.2008
62	4.2099	4.2007
63	4.2097	4.2007
64	4.2096	4.2006
65	4.2094	4.2005
66	4.2093	4.2005
67	4.2092	4.2004
68	4.2090	4.2004
69	4.2089	4.2004
70	4.2088	4.2003
71	4.2087	4.2003
72	4.2085	4.2003
73	4.2084	4.2002
74	4.2083	4.2002
75	4.2082	4.2002

CALCULATED SMITH-WILSON ZERO COUPON CURVES - SMITH WILSON INTERPOLATION

EUR			EUR			UK			UK			US			US		
12/30/2011			12/31/2010			12/30/2011			12/31/2010			12/30/2011			12/31/2010		
Maturity	Curve	Forward	Maturity	Curve	Forward	Maturity	Curve	Forward	Maturity	Curve	Forward	Maturity	Curve	Forward	Maturity	Curve	Forward
76	3.6108	4.1947	76	4.0109	4.1981	76	3.4392	4.1548	76	3.7716	4.1566	76	3.4084	4.1872	76	4.2081	4.2002
77	3.6184	4.1968	77	4.0133	4.1983	77	3.4485	4.1610	77	3.7767	4.1607	77	3.4185	4.1906	77	4.2080	4.2002
78	3.6258	4.1956	78	4.0157	4.1989	78	3.4576	4.1629	78	3.7817	4.1655	78	3.4283	4.1894	78	4.2079	4.2001
79	3.6330	4.1960	79	4.0180	4.1985	79	3.4666	4.1664	79	3.7865	4.1678	79	3.4379	4.1904	79	4.2078	4.2001
80	3.6400	4.1963	80	4.0202	4.1987	80	3.4753	4.1696	80	3.7913	4.1709	80	3.4473	4.1913	80	4.2077	4.2001
81	3.6469	4.1982	81	4.0225	4.1988	81	3.4839	4.1744	81	3.7961	4.1736	81	3.4565	4.1941	81	4.2076	4.2001
82	3.6536	4.1969	82	4.0246	4.1994	82	3.4923	4.1750	82	3.8007	4.1772	82	3.4655	4.1928	82	4.2075	4.2001
83	3.6601	4.1972	83	4.0267	4.1990	83	3.5006	4.1773	83	3.8052	4.1784	83	3.4742	4.1934	83	4.2074	4.2001
84	3.6665	4.1974	84	4.0288	4.1991	84	3.5086	4.1795	84	3.8097	4.1804	84	3.4827	4.1940	84	4.2073	4.2001
85	3.6727	4.1991	85	4.0308	4.1992	85	3.5165	4.1832	85	3.8141	4.1823	85	3.4911	4.1965	85	4.2073	4.2001
86	3.6788	4.1978	86	4.0327	4.1997	86	3.5242	4.1831	86	3.8184	4.1849	86	3.4993	4.1950	86	4.2072	4.2001
87	3.6848	4.1980	87	4.0346	4.1993	87	3.5318	4.1847	87	3.8226	4.1854	87	3.5072	4.1954	87	4.2071	4.2001
88	3.6906	4.1982	88	4.0365	4.1993	88	3.5392	4.1861	88	3.8267	4.1868	88	3.5151	4.1958	88	4.2070	4.2001
89	3.6963	4.1983	89	4.0383	4.1994	89	3.5465	4.1874	89	3.8308	4.1880	89	3.5227	4.1962	89	4.2069	4.2001
90	3.7019	4.1984	90	4.0401	4.1995	90	3.5536	4.1886	90	3.8347	4.1892	90	3.5302	4.1965	90	4.2069	4.2000
91	3.7073	4.1986	91	4.0419	4.1995	91	3.5606	4.1896	91	3.8386	4.1902	91	3.5375	4.1968	91	4.2068	4.2000
92	3.7126	4.1987	92	4.0436	4.1995	92	3.5674	4.1906	92	3.8425	4.1911	92	3.5446	4.1971	92	4.2067	4.2000
93	3.7179	4.2001	93	4.0453	4.1996	93	3.5741	4.1932	93	3.8462	4.1919	93	3.5516	4.1991	93	4.2066	4.2000
94	3.7230	4.1989	94	4.0469	4.2000	94	3.5807	4.1922	94	3.8499	4.1936	94	3.5585	4.1975	94	4.2066	4.2000
95	3.7280	4.1989	95	4.0485	4.1996	95	3.5871	4.1929	95	3.8535	4.1933	95	3.5652	4.1977	95	4.2065	4.2000
96	3.7329	4.1990	96	4.0501	4.1997	96	3.5934	4.1936	96	3.8571	4.1940	96	3.5718	4.1979	96	4.2064	4.2000
97	3.7377	4.2004	97	4.0516	4.1997	97	3.5996	4.1958	97	3.8605	4.1945	97	3.5782	4.1998	97	4.2064	4.2000
98	3.7424	4.1991	98	4.0531	4.2001	98	3.6056	4.1947	98	3.8640	4.1960	98	3.5845	4.1982	98	4.2063	4.2000
99	3.7470	4.1992	99	4.0546	4.1997	99	3.6116	4.1951	99	3.8673	4.1955	99	3.5907	4.1984	99	4.2062	4.2000
100	3.7515	4.1993	100	4.0561	4.1997	100	3.6174	4.1956	100	3.8706	4.1959	100	3.5968	4.1985	100	4.2062	4.2000
101	3.7559	4.2005	101	4.0575	4.1998	101	3.6231	4.1976	101	3.8738	4.1963	101	3.6027	4.2003	101	4.2061	4.2000
102	3.7603	4.1993	102	4.0589	4.2002	102	3.6287	4.1963	102	3.8770	4.1975	102	3.6085	4.1987	102	4.2061	4.2000
103	3.7645	4.1994	103	4.0603	4.1998	103	3.6342	4.1966	103	3.8801	4.1969	103	3.6143	4.1988	103	4.2060	4.2000
104	3.7687	4.1994	104	4.0616	4.1998	104	3.6396	4.1969	104	3.8831	4.1972	104	3.6199	4.1989	104	4.2059	4.2000
105	3.7728	4.2006	105	4.0629	4.1998	105	3.6449	4.1987	105	3.8861	4.1974	105	3.6254	4.2005	105	4.2059	4.2000
106	3.7768	4.1995	106	4.0642	4.2002	106	3.6501	4.1974	106	3.8890	4.1985	106	3.6308	4.1990	106	4.2058	4.2000
107	3.7808	4.1995	107	4.0655	4.1998	107	3.6552	4.1976	107	3.8919	4.1979	107	3.6361	4.1991	107	4.2058	4.2000
108	3.7846	4.1995	108	4.0667	4.1998	108	3.6603	4.1978	108	3.8948	4.1981	108	3.6413	4.1991	108	4.2057	4.2000
109	3.7884	4.2007	109	4.0679	4.1998	109	3.6652	4.1995	109	3.8975	4.1982	109	3.6464	4.2007	109	4.2057	4.2000
110	3.7922	4.1996	110	4.0691	4.2002	110	3.6700	4.1981	110	3.9003	4.1992	110	3.6514	4.1992	110	4.2056	4.2000
111	3.7958	4.1996	111	4.0703	4.1999	111	3.6748	4.1983	111	3.9030	4.1985	111	3.6563	4.1993	111	4.2056	4.2000
112	3.7994	4.1996	112	4.0715	4.1999	112	3.6794	4.1984	112	3.9056	4.1986	112	3.6612	4.1993	112	4.2055	4.2000
113	3.8030	4.2007	113	4.0726	4.1999	113	3.6840	4.2000	113	3.9082	4.1987	113	3.6659	4.2008	113	4.2055	4.2000
114	3.8064	4.1996	114	4.0737	4.2002	114	3.6885	4.1987	114	3.9107	4.1997	114	3.6706	4.1994	114	4.2054	4.2000
115	3.8099	4.1996	115	4.0748	4.1999	115	3.6930	4.1988	115	3.9132	4.1989	115	3.6752	4.1994	115	4.2054	4.2000
116	3.8132	4.1996	116	4.0759	4.1999	116	3.6973	4.1988	116	3.9157	4.1990	116	3.6797	4.1994	116	4.2053	4.2000
117	3.8165	4.2007	117	4.0770	4.1999	117	3.7016	4.2003	117	3.9181	4.1991	117	3.6841	4.2009	117	4.2053	4.2000
118	3.8198	4.1997	118	4.0780	4.2002	118	3.7058	4.1990	118	3.9205	4.2000	118	3.6885	4.1995	118	4.2052	4.2000
119	3.8230	4.1997	119	4.0790	4.1999	119	3.7099	4.1991	119	3.9228	4.1992	119	3.6928	4.1995	119	4.2052	4.2000
120	3.8261	4.1997	120	4.0800	4.1999	120	3.7140	4.1991	120	3.9251	4.1993	120	3.6970	4.1995	120	4.2051	4.2000
121	3.8292	4.2007	121	4.0810	4.1999	121	3.7180	4.2005	121	3.9274	4.1993	121	3.7011	4.2009	121	4.2051	4.2000
122	3.8322	4.1997	122	4.0820	4.2002	122	3.7219	4.1992	122	3.9296	4.2001	122	3.7052	4.1995	122	4.2051	4.2000
123	3.8352	4.1997	123	4.0830	4.1999	123	3.7258	4.1993	123	3.9318	4.1994	123	3.7092	4.1996	123	4.2050	4.2000
124	3.8381	4.1997	124	4.0839	4.1999	124	3.7296	4.1993	124	3.9340	4.1995	124	3.7132	4.1996	124	4.2050	4.2000
125	3.8410	4.2007	125	4.0848	4.1999	125	3.7334	4.2007	125	3.9361	4.1995	125	3.7171	4.2009	125	4.2049	4.2000
126	3.8439	4.1997	126	4.0857	4.2002	126	3.7371	4.1994	126	3.9382	4.2003	126	3.7209	4.1996	126	4.2049	4.2000
127	3.8467	4.1997	127	4.0866	4.1999	127	3.7407	4.1994	127	3.9403	4.1996	127	3.7246	4.1996	127	4.2049	4.2000
128	3.8494	4.1997	128	4.0875	4.1999	128	3.7443	4.1994	128	3.9423	4.1996	128	3.7283	4.1996	128	4.2048	4.2000
129	3.8521	4.2007	129	4.0884	4.1999	129	3.7478	4.2007	129	3.9443	4.1996	129	3.7320	4.2009	129	4.2048	4.2000
130	3.8548	4.1997	130	4.0893	4.2002	130	3.7513	4.1995	130	3.9462	4.2003	130	3.7356	4.1996	130	4.2048	4.2000
131	3.8574	4.1997	131	4.0901	4.1999	131	3.7547	4.1995	131	3.9482	4.1997	131	3.7391	4.1996	131	4.2047	4.2000
132	3.8600	4.1998	132	4.0909	4.1999	132	3.7581	4.1995	132	3.9501	4.1997	132	3.7426	4.1996	132	4.2047	4.2000
133	3.8626	4.2007	133	4.0918	4.1999	133	3.7614	4.2008	133	3.9520	4.1997	133	3.7460	4.2009	133	4.2046	4.2000
134	3.8651	4.1998	134	4.0926	4.2002	134	3.7646	4.1996	134	3.9538	4.2004	134	3.7494	4.1997	134	4.2046	4.2000
135	3.8676	4.1998	135	4.0934	4.1999	135	3.7679	4.1996	135	3.9556	4.1997	135	3.7527	4.1997	135	4.2046	4.2000
136	3.8700	4.1998	136	4.0941	4.1999	136	3.7710	4.1996	136	3.9574	4.1997	136	3.7560	4.1997	136	4.2045	4.2000
137	3.8724	4.2007	137	4.0949	4.1999	137	3.7742	4.2008	137	3.9592	4.1997	137	3.7593	4.2009	137	4.2045	4.2000
138	3.8748	4.1998	138	4.0957	4.2002	138	3.7772	4.1996	138	3.9609	4.2004	138	3.7624	4.1997	138	4.2045	4.2000
139	3.8771	4.1998	139	4.0964	4.1999	139	3.7803	4.1996	139	3.9626	4.1998	139	3.7656	4.1997	139	4.2044	4.2000
140	3.8794	4.1998	140	4.0972	4.1999	140	3.7833	4.1996	140	3.9643	4.1998	140	3.7687	4.1997	140	4.2044	4.2000
141	3.8817	4.2007	141	4.0979	4.1999	141	3.7862	4.2008	141	3.9660	4.1998	141	3.7717	4.2009	141	4.2044	4.2000
142	3.8839	4.1998	142	4.0986	4.2002	142	3.7891	4.1997	142	3.9676	4.2004	142	3.7747	4.1997	142	4.2044	4.2000
143	3.8861	4.1998	143	4.0993	4.1999	143	3.7920	4.1997	143	3.9693	4.1998	143	3.7777	4.1997	143	4.2043	4.2000
144	3.8883	4.1998	144	4.1000	4.1999	144	3.7948	4.1997	144	3.9709	4.1998	144	3.7806	4.1997	144	4.2043	4.2000
145	3.8905	4.2006	145	4.1007	4.1999	145	3.7976	4.2008	145	3.9724	4.1998	145	3.7835	4.2009	145	4.2043	4.2000
146	3.8926	4.1998	146	4.1014	4.2002	146	3.8004	4.1997	146	3.9740	4.2004	146	3.7864	4.1997	146	4.2042	4.2000
147	3.8947	4.1998	147	4.1021	4.1999	147	3.8031	4.1997	147	3.9755	4.1998	147	3.7892	4.1997	147	4.2042	4.2000
148	3.8967	4.1998	148	4.1027	4.1999	148	3.8057	4.1997	148	3.9771	4.1998	148	3.7920	4.1997	148	4.2042	4.2000
149	3.8988	4.2006	149	4.1034	4.1999	149	3.8084	4.2008	149	3.9786	4.1998	149	3.7947	4.2008	149	4.2041	4.2000
150	3.9008	4.1998	150	4.1040	4.2002	150	3.8110	4.1997	150	3.9800	4.2004	150	3.7974	4.1997	150	4.2041	4.2000